

Precise limits from lepton flavour violating processes on the littlest Higgs model with T-parity

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Precise limits from lepton flavour violating processes on the littlest Higgs model with T-parity

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ABSTRACT: We recalculate the leading one-loop contributions to $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee\bar{e}$ in the Littlest Higgs model with T-parity, recovering previous results for the former. When all the Goldstone interactions are taken into account, the latter is also ultraviolet finite. The present experimental limits on these processes require a somewhat heavy effective scale ~ 2.5 TeV, or the flavour alignment of the Yukawa couplings of light and heavy leptons at the $\sim 10\%$ level, or the splitting of heavy lepton masses to a similar precision. Present limits on τ decays set no bounds on the corresponding parameters involving the τ lepton.

KEYWORDS: Higgs Physics, Rare Decays, Beyond Standard Model.

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1. Introduction

Little Higgs models [1] offer an explanation to the little hierarchy between the Higgs mass M_h assumed to be near the electroweak scale $v = 246$ GeV and the new physics (NP) scale f , whose natural value is expected to be ~ 1 TeV [2]. In contrast with supersymmetry, where the large one-loop Standard Model (SM) contributions to the Higgs mass are cancelled by the contributions from the corresponding supersymmetric partners with masses ~ 1 TeV and spins differing by $\pm 1/2$ (see [3] and references therein), Little Higgs (LH) models stabilize M_h by making the Higgs a pseudo-Goldstone boson of a broken global symmetry. The cancellation is in this case between particles with the same spin belonging to the same multiplets of this approximate symmetry. Which of these SM extensions, if any, is at work will be hopefully established at the LHC [4, 5].

Supersymmetry is linearly realized in the minimal supersymmetric SM (MSSM). This and other simple supersymmetric extensions of the SM have other interesting phenomenological properties, like, for instance, the unification of gauge couplings at very high scales [6], which is not the case for LH models. However, they have no built-in low energy mechanism to explain the observed fermion mass hierarchy or flavour conservation. As a matter of fact, it can be argued that supergravity is phenomenologically relevant [3, 7] because it can provide the necessary initial conditions to explain the precise fine-tuning required among the many new parameters of the MSSM, which otherwise would result in too large flavour changing processes [8]. This has been historically the problem of many SM extensions [9]. If the NP is near the TeV scale, it faces in general the problem of naturally explaining why it is *aligned* with the SM Yukawa interactions, as experimentally required. Although the SM can not explain the large hierarchy between fermion masses, which by the way is several orders of magnitude more demanding than the little hierarchy, it naturally accommodates the absence of flavour changing neutral currents (FCNC) [10]. LH models are not designed to solve the flavour puzzle either, and one must expect stringent constraints on the new parameters involving the heavy sector. The study of FCNC processes in Littlest Higgs models has been addressed in the literature [11]. In this paper we revise the calculation of the decay rates of the lepton flavour violating (LFV) processes $\mu \rightarrow e\gamma$ [12, 13] and $\mu \rightarrow ee\bar{e}$ [13] in the Littlest Higgs model with T-parity (LHT) [14], obtaining an ultraviolet finite result also for the latter.¹ Indeed, when all Goldstone boson interactions of the new leptons are taken into account, the one-loop contributions to the amplitudes are well-defined [16, 17], scaling approximately in the two family case like $(v^2/f^2)\sin 2\theta\delta$, where θ is a measure of the misalignment between the heavy and SM lepton Yukawa couplings and δ is the corresponding heavy lepton mass splitting. As a consequence, the present experimental limits require fine tuning the Yukawa couplings of the new heavy leptons up to 10%, aligning them with their SM counterparts, or making the heavy masses quasi-degenerate. One might also rise the NP scale degrading the motivation of the LH scenario itself. In the general case with three families the new contributions must be tuned to a similar precision but the parameter dependence is more involved. The calculation also applies to τ decays, but the corresponding limits are not restrictive at present. Moreover, it can be easily extended to $\mu - e$ conversion in nuclei [18]. A complete phenomenological analysis comparing as well different LH models will be presented elsewhere.

In LH models the Higgs is a pseudo-Goldstone boson. Thus, M_h is naturally small as long as the new scale f is relatively low, because one expects that cancellations are only protected to one loop and for the dominant contributions. Hence, $4\pi f$ can not be much larger than 10 TeV if we do not wish to invoke some fine tuning again. However, as the model introduces heavy particles the new one-loop contributions to electroweak observables may require rising f significantly above 1 TeV in the absence of model dependent cancellations, in order to be consistent with present electroweak precision data (EWPD) [19]. The LHT is an economical realization of the LH scenario with the further virtue of keeping the new

¹See also [15] which appeared when we were preparing this manuscript. There the cancellation of ultraviolet divergences in the LHT model is also shown, flavour violation in the quark sector is explored and the phenomenology of $K \rightarrow \pi\nu\bar{\nu}$ is analyzed.

contributions to EWPD small. It incorporates a discrete symmetry under which the new particles are odd and the SM ones even. Then all vertices must have an even number of new particles, if any. Similarly to the R symmetry in supersymmetric models, the T symmetry allows us to weaken the experimental limit on the LH effective scale below the TeV [20]. This symmetry also makes stable the lightest T-odd particle, offering, like R-parity does in the supersymmetric case, an alternative candidate for cold dark matter [21].

Nevertheless, as already emphasized these models are not a priori designed to deal with the flavour problem. Therefore, it is important to investigate the constraints on the model parameters implied by the stringent experimental limits on FCNC. We follow an operational approach and calculate the leading contributions to $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee\bar{e}$ in the LHT, recovering previous results for the former [12, 13] but an ultraviolet finite result for the latter. We focus on these processes because the lepton sector is free from large strong corrections, and the experimental limits are quite demanding. In section 2 we review the LHT model to introduce the notation and the Feynman rules needed. The one-loop amplitudes of the LFV processes $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee\bar{e}$ are discussed in section 3. The calculation is straightforward but cumbersome, requiring a careful bookkeeping of the different terms. In section 4 we present the numerical results discussing the dependence on the different parameters of the model. Finally, section 5 is devoted to conclusions, where we also briefly comment on τ decays.

2. The littlest Higgs model with T-parity

2.1 The Lagrangian

The LHT is a non-linear σ model based on the coset space $SU(5)/SO(5)$, with the $SU(5)$ global symmetry broken by the vacuum expectation value (VEV) of a 5×5 symmetric tensor,

$$\Sigma_0 = \begin{pmatrix} \mathbf{0}_{2 \times 2} & 0 & \mathbf{1}_{2 \times 2} \\ 0 & 1 & 0 \\ \mathbf{1}_{2 \times 2} & 0 & \mathbf{0}_{2 \times 2} \end{pmatrix}. \quad (2.1)$$

The 10 unbroken generators T^a , which leave invariant Σ_0 and then satisfy $T^a \Sigma_0 + \Sigma_0 (T^a)^T = 0$, expand the $SO(5)$ algebra; whereas the 14 broken generators X^a , which fulfill $X^a \Sigma_0 - \Sigma_0 (X^a)^T = 0$, expand the Goldstone fields $\Pi = \pi^a X^a$ parameterized as

$$\Sigma(x) = e^{i\Pi/f} \Sigma_0 e^{i\Pi^T/f} = e^{2i\Pi/f} \Sigma_0, \quad (2.2)$$

where f is the effective NP scale. Only the $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$ subgroup of the $SU(5)$ global symmetry is gauged. It is generated by

$$Q_1^a = \frac{1}{2} \begin{pmatrix} \sigma^a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{0}_{2 \times 2} \end{pmatrix}, \quad Y_1 = \frac{1}{10} \text{diag}(3, 3, -2, -2, -2), \quad (2.3)$$

$$Q_2^a = \frac{1}{2} \begin{pmatrix} \mathbf{0}_{2 \times 2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sigma^{a*} \end{pmatrix}, \quad Y_2 = \frac{1}{10} \text{diag}(2, 2, 2, -3, -3), \quad (2.4)$$

with σ^a the three Pauli matrices. The VEV in eq. (2.1) breaks this gauge group down to the SM gauge group $SU(2)_L \times U(1)_Y$, generated by the combinations $\{Q_1^a + Q_2^a, Y_1 + Y_2\} \subset \{T^a\}$. The orthogonal combinations are a subset of the broken generators, $\{Q_1^a - Q_2^a, Y_1 - Y_2\} \subset \{X^a\}$. Thus, the Goldstone fields

$$\Pi = \begin{pmatrix} -\frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & -\frac{\omega^+}{\sqrt{2}} & -i\frac{\pi^+}{\sqrt{2}} & -i\Phi^{++} & -i\frac{\Phi^+}{\sqrt{2}} \\ -\frac{\omega^-}{\sqrt{2}} & \frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & \frac{v+h+i\pi^0}{2} & -i\frac{\Phi^+}{\sqrt{2}} & \frac{-i\Phi^0 + \Phi^P}{\sqrt{2}} \\ i\frac{\pi^-}{\sqrt{2}} & \frac{v+h-i\pi^0}{2} & \sqrt{\frac{4}{5}}\eta & -i\frac{\pi^+}{\sqrt{2}} & \frac{v+h+i\pi^0}{2} \\ i\Phi^{--} & i\frac{\Phi^-}{\sqrt{2}} & i\frac{\pi^-}{\sqrt{2}} & -\frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & -\frac{\omega^-}{\sqrt{2}} \\ i\frac{\Phi^-}{\sqrt{2}} & \frac{i\Phi^0 + \Phi^P}{\sqrt{2}} & \frac{v+h-i\pi^0}{2} & -\frac{\omega^+}{\sqrt{2}} & \frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} \end{pmatrix} \quad (2.5)$$

decompose into the SM Higgs doublet $(-i\pi^+/\sqrt{2}, (v+h+i\pi^0)/2)^T$, a complex $SU(2)_L$ triplet Φ , and the longitudinal modes of the heavy gauge fields ω^\pm, ω^0 and η .²

As emphasized in the previous section, we can make the new contributions to electroweak precision observables small enough introducing a T-parity under which the SM particles are even and the new particles are odd. An obvious choice for the action of such T-parity on the gauge fields G_i is the exchange of the gauge subgroups $[SU(2) \times U(1)]_1$ and $[SU(2) \times U(1)]_2$,

$$G_1 \xrightarrow{T} G_2. \quad (2.6)$$

Then, T invariance requires that the gauge couplings associated to both factors are equal. This leaves the following gauge Lagrangian unchanged,

$$\mathcal{L}_G = \sum_{j=1}^2 \left[-\frac{1}{2} \text{Tr} \left(\widetilde{W}_{j\mu\nu} \widetilde{W}_j^{\mu\nu} \right) - \frac{1}{4} B_{j\mu\nu} B_j^{\mu\nu} \right], \quad (2.7)$$

where

$$\widetilde{W}_{j\mu} = W_{j\mu}^a Q_j^a, \quad \widetilde{W}_{j\mu\nu} = \partial_\mu \widetilde{W}_{j\nu} - \partial_\nu \widetilde{W}_{j\mu} - ig \left[\widetilde{W}_{j\mu}, \widetilde{W}_{j\nu} \right], \quad B_{j\mu\nu} = \partial_\mu B_{j\nu} - \partial_\nu B_{j\mu}. \quad (2.8)$$

(Summation over index a , which runs on the corresponding $SU(2)$ generators, is always assumed when repeated.) The T-even combinations multiplying the unbroken gauge generators correspond to the SM gauge bosons,

$$W^\pm = \frac{1}{2} [(W_1^1 + W_2^1) \mp i(W_1^2 + W_2^2)], \quad W^3 = \frac{W_1^3 + W_2^3}{\sqrt{2}}, \quad B = \frac{B_1 + B_2}{\sqrt{2}}, \quad (2.9)$$

whereas the T-odd combinations

$$W_H^\pm = \frac{1}{2} [(W_1^1 - W_2^1) \mp i(W_1^2 - W_2^2)], \quad W_H^3 = \frac{W_1^3 - W_2^3}{\sqrt{2}}, \quad B_H = \frac{B_1 - B_2}{\sqrt{2}}, \quad (2.10)$$

²In the following we use for the SM fields and couplings the conventions in ref. [22]. In particular, $\phi^+ = -i\pi^+$, $\phi^0 = \pi^0$.

expand the heavy gauge sector.

In order to ensure that the SM Higgs doublet is T-even and the remaining Goldstone fields are T-odd, the T action on the scalar fields is defined as follows,

$$\Pi \xrightarrow{T} -\Omega\Pi\Omega, \quad \Omega = \text{diag}(-1, -1, 1, -1, -1), \quad (2.11)$$

where Ω is an element of the center of the gauge group,³ which commutes with Σ_0 but not with the full global symmetry. Then,

$$\Sigma \xrightarrow{T} \tilde{\Sigma} = \Omega\Sigma_0\Sigma^\dagger\Sigma_0\Omega, \quad (2.12)$$

and the scalar Lagrangian

$$\mathcal{L}_S = \frac{f^2}{8} \text{Tr} \left[(D_\mu \Sigma)^\dagger (D^\mu \Sigma) \right], \quad (2.13)$$

with

$$D_\mu \Sigma = \partial_\mu \Sigma - \sqrt{2}i \sum_{j=1}^2 [gW_{j\mu}^a (Q_j^a \Sigma + \Sigma Q_j^{aT}) - g' B_{j\mu} (Y_j \Sigma + \Sigma Y_j^T)], \quad (2.14)$$

is also gauge and T-invariant.

This discrete symmetry must be implemented in the fermion sector too. This is less straightforward. In fact, there is no proposed model fulfilling the three desired conditions: to give masses to all (SM) fermions with Yukawa couplings, preserving a discrete symmetry under which all new particles are odd and the SM ones even, and keeping the full global symmetry before introducing the symmetry breaking. Although terms explicitly breaking the global symmetries at the Lagrangian level must manifest as *badly* behaved contributions to physical processes [17], this will not be our case since all the explicit couplings entering in the calculation we are interested in can be derived from Lagrangian terms which are symmetric. Following refs. [23, 24] we introduce two left-handed fermion doublets in incomplete SU(5) multiplets, one transforming just under SU(2)₁ and the other under SU(2)₂, for each SM left-handed lepton doublet:

$$\Psi_1 = \begin{pmatrix} -i\sigma^2 l_{1L} \\ 0 \\ 0 \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} 0 \\ 0 \\ -i\sigma^2 l_{2L} \end{pmatrix}, \quad (2.15)$$

where $l_{iL} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}$, $i = 1, 2$, and

$$\Psi_1 \longrightarrow V^* \Psi_1, \quad \Psi_2 \longrightarrow V \Psi_2, \quad (2.16)$$

under an SU(5) transformation V . We define the T-parity action on these fermions

$$\Psi_1 \xleftrightarrow{T} \Omega \Sigma_0 \Psi_2. \quad (2.17)$$

³Note that we have reversed the sign of Ω as compared to the literature, to make it a group element.

Then the usual T-even combination $\Psi_1 + \Omega \Sigma_0 \Psi_2$ remains light and is identified, up to the proper normalization, with the SM fermion doublet. The T-odd combination $\Psi_1 - \Omega \Sigma_0 \Psi_2$ pairs with a right-handed doublet (eigenvector of T), in a complete SO(5) multiplet,

$$\Psi_R = \begin{pmatrix} \cdot \\ \cdot \\ -i\sigma^2 l_{HR} \end{pmatrix}, \quad \Psi_R \xrightarrow{T} \Omega \Psi_R, \quad \Psi_R \longrightarrow U \Psi_R, \quad (2.18)$$

where U is an SO(5) transformation defined below, to form a heavy Dirac doublet. With this aim in mind, a non-linear Yukawa Lagrangian is introduced,

$$\mathcal{L}_{Y_H} = -\kappa f \left(\bar{\Psi}_2 \xi + \bar{\Psi}_1 \Sigma_0 \xi^\dagger \right) \Psi_R + \text{h.c.}, \quad (2.19)$$

where $\xi = e^{i\Pi/f}$. This is indeed T-invariant, since eq. (2.11) implies

$$\xi \xrightarrow{T} \Omega \xi^\dagger \Omega, \quad (2.20)$$

and invariant under global transformations,

$$\Sigma = \xi^2 \Sigma_0 \longrightarrow V \Sigma V^T \quad \Rightarrow \quad \xi \longrightarrow V \xi U^\dagger \equiv U \xi \Sigma_0 V^T \Sigma_0, \quad (2.21)$$

where V is the global SU(5) transformation and U a function of V and Π taking values in the Lie algebra of the unbroken SO(5). It must be noted that the gauge singlet χ_R , completing the SO(5) representation

$$\Psi_R = \begin{pmatrix} \tilde{\psi}_R \\ \chi_R \\ -i\sigma^2 l_{HR} \end{pmatrix} \quad (2.22)$$

and assumed to be heavy, is T-even.⁴ On the other hand, the extra doublet $\tilde{\psi}_R$, which is also assumed to be heavy enough to agree with EWPD, is T-odd as desired.

We have just introduced all heavy fields we need. However, one important comment is in order. The Yukawa-type Lagrangian \mathcal{L}_{Y_H} fixes the transformation properties of the heavy fermions and then their gauge couplings, in particular the non-linear couplings of the right-handed heavy fermions [25],

$$\begin{aligned} \mathcal{L}_F = & i\bar{\Psi}_1 \gamma^\mu D_\mu^* \Psi_1 + i\bar{\Psi}_2 \gamma^\mu D_\mu \Psi_2 \\ & + i\bar{\Psi}_R \gamma^\mu \left(\partial_\mu + \frac{1}{2} \xi^\dagger (D_\mu \xi) + \frac{1}{2} \xi (\Sigma_0 D_\mu^* \Sigma_0 \xi^\dagger) \right) \Psi_R \end{aligned} \quad (2.23)$$

with

$$D_\mu = \partial_\mu - \sqrt{2}ig(W_{1\mu}^a Q_1^a + W_{2\mu}^a Q_2^a) + \sqrt{2}ig'(Y_1 B_{1\mu} + Y_2 B_{2\mu}). \quad (2.24)$$

⁴If we had defined the T action on the fermions $\Psi_1 \xrightarrow{T} -\Sigma_0 \Psi_2$, $\Psi_R \xrightarrow{T} -\Psi_R$ and the Yukawa Lagrangian with Ω 's, $\mathcal{L}_{Y_H} = -\kappa f (\bar{\Psi}_2 \xi + \bar{\Psi}_1 \Sigma_0 \Omega \xi^\dagger \Omega) \Psi_R + \text{h.c.}$, all new fermions would be T-odd and the new Lagrangian invariant under the new T-parity [24], but not under the full global symmetry because Ω does not commute with SU(5) neither with SO(5), although it does commute with the gauge group. We must insist that the explicit couplings entering in our calculation are the same in both cases.

The Lagrangian of eq. (2.23) includes the proper $O(v^2/f^2)$ couplings to Goldstone fields, absent in [13, 26], that render the one-loop amplitudes ultraviolet finite. Besides, in order to assign the proper SM hypercharge $y = -1$ to the charged right-handed leptons ℓ_R , which are SU(5) singlets and T-even, one can enlarge SU(5) with two extra U(1) groups, since otherwise their hypercharge would be zero. Then, the corresponding gauge and T invariant Lagrangian reads

$$\mathcal{L}'_F = i\bar{\ell}_R\gamma^\mu(\partial_\mu + ig'yB_\mu)\ell_R. \tag{2.25}$$

For the lepton sector and the calculation we are interested in these are all the necessary Lagrangian terms. However, in order to define what a muon or an electron is, we have to diagonalise the mass matrix $(M_\ell)_{ij} = (\lambda_\ell)_{ij}v$ in the corresponding Yukawa Lagrangian \mathcal{L}_Y which we assume to have all required properties [25, 27]⁵ (and also to include light neutrino masses). This gives to leading order a mass term for the charged leptons $m_{\ell i}\bar{\ell}_L^i\ell_R^i + \text{h.c.}$, with

$$m_{\ell i'}\delta_{i'j'} = (V_L^{\ell\dagger})_{i'i}(\lambda_\ell)_{ij}(V_R^\ell)_{jj'}v \tag{2.26}$$

and $V_{L,R}^\ell$ two unitary matrices.⁶

Finally, in order to perform the calculation in the mass eigenstate basis we have to diagonalise the full Lagrangian

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_{Y_H} + \mathcal{L}_F + \mathcal{L}'_F + \mathcal{L}_Y, \tag{2.27}$$

and reexpress it in the mass eigenstate basis. The corresponding masses and eigenvectors up to order v^2/f^2 are given in appendix A. The Feynman rules are collected in appendix B. They are obtained expanding \mathcal{L} to the required order. The coupling overlooked in [13] is the v^2/f^2 correction to the right-handed coupling g_R of the $Z\bar{\nu}_H^i\nu_H^j$ vertex, resulting from the expansion of the last two terms of \mathcal{L}_F in eq. (2.23).

2.2 Flavour mixing

The new contributions to charged LFV processes must be proportional to the ratio of the electroweak and the LHT breaking scales v^2/f^2 and to a combination of the matrix

⁵Right-handed leptons, as the other right-handed SM fermions, are usually taken to be singlets under the non-abelian symmetries, transforming only under the gauge abelian subgroup. We must note that this may be a too strong assumption. If we want to couple them to their left-handed counterpart, one may be inspired by the following observation. There is only one SU(5) singlet in the decomposition of the product of two Σ 's and one left-handed fermion multiplet, $\sum_{\alpha_i=1}^5 \epsilon^{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} [(\Sigma)_{\alpha_1\alpha_2}(\Sigma)_{\alpha_3\alpha_4}\Psi_{2\alpha_5} + (\Sigma^\dagger)_{\alpha_1\alpha_2}(\Sigma^\dagger)_{\alpha_3\alpha_4}\Psi_{1\alpha_5}]$, where $\epsilon^{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5}$ is the totally antisymmetric tensor and the second term is the T transformed of the first one. Alternatively, one could multiply three Σ 's and the other left-handed fermion multiplet, $\sum_{\alpha_i=1}^5 \epsilon^{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} \delta^{\alpha_6\alpha_7} [(\Sigma)_{\alpha_1\alpha_2}(\Sigma)_{\alpha_3\alpha_4}(\Sigma)_{\alpha_5\alpha_6}\Psi_{1\alpha_7} + (\Sigma^\dagger)_{\alpha_1\alpha_2}(\Sigma^\dagger)_{\alpha_3\alpha_4}(\Sigma^\dagger)_{\alpha_5\alpha_6}\Psi_{2\alpha_7}]$, with $\delta^{\alpha_6\alpha_7}$ the Kronecker delta. In both cases, we get the wrong Higgs coupling. This is so because this product is an SU(5) singlet and then the Higgs coupling reads $i\pi^+l^- + (v+h+i\pi^0)\nu/\sqrt{2}$. (In these expressions there are neither Ω 's nor Σ_0 's because the determinant of Ω is 1 and $\epsilon^{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5}(\Sigma_0)_{\alpha_1\beta_1}(\Sigma_0)_{\alpha_2\beta_2}(\Sigma_0)_{\alpha_3\beta_3}(\Sigma_0)_{\alpha_4\beta_4}(\Sigma_0)_{\alpha_5\beta_5} = \epsilon^{\beta_1\beta_2\beta_3\beta_4\beta_5}$.) Then, getting the correct coupling $(v+h-i\pi^0)l^-/\sqrt{2} + i\pi^-\nu$ requires the explicit breaking of SU(5). If ξ is introduced in the game, one eventually has to break SO(5) as well.

⁶We denote the mass eigenstates with primes when necessary to distinguish them from the current eigenstates.

elements describing the misalignment of the heavy and charged lepton Yukawa couplings. Let us then set our conventions for the description of the heavy-light mixing relevant to our analysis, and in particular to the Feynman rules discussed above and collected in appendix B. The SM interaction and mass eigenstates are related by the unitary matrices in eq. (2.26),

$$\ell_L = V_L^\ell \ell'_L, \quad \ell_R = V_R^\ell \ell'_R. \quad (2.28)$$

Then the SM charged current Lagrangian reads

$$\mathcal{L}_{CC}^{\text{SM}} = -\frac{g}{\sqrt{2}} \bar{\nu}_L \not{W}^\dagger \ell_L + \text{h.c.} = -\frac{g}{\sqrt{2}} \bar{\nu}'_L V_L^{\nu\dagger} V_L^\ell \not{W}^\dagger \ell'_L + \text{h.c.}, \quad (2.29)$$

where we have also introduced the corresponding rotation for the neutrinos. Thus, only the combination $V_{\text{PMNS}}^\dagger = V_L^{\nu\dagger} V_L^\ell$ is observable. It must be noted, however, that the neutrino contributions to LFV processes are negligible in the SM because so are their masses. Hence, V_L^ν can be assumed to be unity. Similarly we can also diagonalise the heavy Yukawa couplings in eq. (2.19),

$$m_{i'j}^H \delta_{i'j'} = (V_L^{H\dagger})_{i'i} \kappa_{ij} (V_R^H)_{jj'} \sqrt{2} f, \quad (2.30)$$

where V_L^H acts on the left-handed fields and V_R^H acts on the right-handed fields. Note that there is no distinction between up- and down-type leptons. The T-odd gauge boson interactions arising from the corresponding kinetic terms for left-handed leptons in eq. (2.23) are proportional to

$$\bar{l}_{L-} \not{G}_- l_{L+} + \text{h.c.} = \bar{l}_{HL} V_L^{H\dagger} \not{G}_H \begin{pmatrix} V_L^\nu \nu_L \\ V_L^\ell \ell_L \end{pmatrix} + \text{h.c.} \quad (2.31)$$

where G_- and l_{L-} are the heavy, T-odd gauge bosons and fermions and l_{L+} are the SM, T-even fermions in the interaction basis, whereas $G_H = A_H, Z_H, W_H$; $l_H = (\nu_H, \ell_H)^T$; and ν_L and ℓ_L are the corresponding mass eigenstates. Then, in analogy with the PMNS matrix, the observable rotations are now

$$V_{H\nu} \equiv V_L^{H\dagger} V_L^\nu, \quad V_{H\ell} \equiv V_L^{H\dagger} V_L^\ell. \quad (2.32)$$

Note that both matrices are related, $V_{H\nu}^\dagger V_{H\ell} = V_{\text{PMNS}}^\dagger$ [28], but this relation can not be tested unless $V_{H\nu}$ can be measured. The new contributions to the LFV amplitudes describing a muon decay to an electron are then proportional to $V_{H\ell}^{ie*} V_{H\ell}^{i\mu}$, with i counting the heavy lepton doublets.

3. New contributions to LFV processes

As noted above, the SM contributions to the LFV processes $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee\bar{e}$ are negligible for they are proportional to the observed neutrino masses. On the other hand the new LHT contributions can be a priori large. In particular, one expects that the dominant

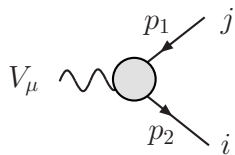


Figure 1: Effective vector-fermion vertex.

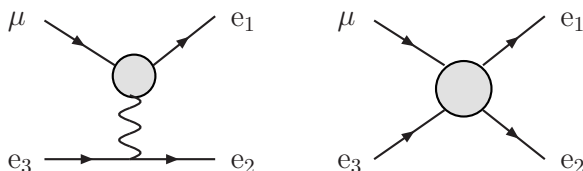


Figure 2: Generic penguin and box diagrams for $\mu \rightarrow e_1 e_2 \bar{e}_3$. Crossed diagrams with e_1 and e_2 exchanged must be added.

contributions come from the exchange of the new vector bosons and heavy fermions required to realise the discrete symmetry T .⁷ Here we study both processes in turn.

The amplitude $\mu \rightarrow e\gamma$ is proportional to the vertex in figure 1, whose most general structure for on-shell fermions $f_{i,j}$ can be written in terms of six form factors:

$$i\Gamma^\mu(p_1, p_2) = ie[\gamma^\mu(F_L^V P_L + F_R^V P_R) + (iF_M^V + F_E^V \gamma_5)\sigma^{\mu\nu}Q_\nu + (iF_S^V + F_P^V \gamma_5)Q^\mu], \quad (3.1)$$

with $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$ and $Q = p_2 - p_1$ the vector boson momentum entering into the vertex. If the vector boson V is a photon, the $U(1)$ gauge symmetry is unbroken and current conservation implies

$$(m_i - m_j)(F_L^\gamma + F_R^\gamma) + 2iQ^2 F_S^\gamma = 0, \quad (3.2)$$

$$-(m_i + m_j)(F_L^\gamma - F_R^\gamma) + 2Q^2 F_P^\gamma = 0. \quad (3.3)$$

Hence, the LFV process $f_j \rightarrow f_i\gamma$ with $i \neq j$ where the photon is on-shell ($Q^2 = 0$) is completely described by a dipole transition. Indeed, according to eqs. (3.2), (3.3) $F_L^\gamma = F_R^\gamma = 0$ for on-shell photons, while the form factors $F_{S,P}^\gamma$ do not contribute to the amplitude because real photons are transverse. Then, the total width for $\ell_j \rightarrow \ell_i\gamma$ is given by [29, 32–35]

$$\Gamma(\ell_j \rightarrow \ell_i\gamma) = \frac{\alpha}{2} m_{\ell_j}^3 (|F_M^\gamma|^2 + |F_E^\gamma|^2). \quad (3.4)$$

On the other hand, two types of diagrams contribute to $\mu \rightarrow ee\bar{e}$ (see figure 2). Now, in the diagrams of the first type (penguins) the exchanged gauge boson V can be a γ

⁷The addition of new vector-like leptons in general imply large FCNC already at tree level [10, 29], and stringent constraints from EWPD and LFV processes [30]. In the LHT they are absent because T -parity forbids the coupling of a SM gauge boson to one light and one heavy fermion. Analogously, the presence of heavy scalar triplets with hypercharge 1 in general allows for their direct coupling to two (SM) lepton doublets (for a review and further references see [5, 31]). This is also absent in the LHT because the triplet Φ in eq. (2.5) is T -odd and the SM leptons are T -even.

or a Z but not a heavy vector boson for the coupling is forbidden by T-parity. (Higgs-penguins are neglected.) F_L^V and F_R^V do not vanish in penguins. In fact, for γ these form factors are proportional to Q^2 , as we have explicitly checked. Besides, as the gauge boson couples to two on-shell electrons, the contributions from $F_{S,P}^V$ are irrelevant for they multiply the electron mass. The amplitude for this process also receives contributions from box diagrams. The total amplitude for $\mu(p) \rightarrow e(p_1) e(p_2) \bar{e}(p_3)$ can be then written [32]

$$\mathcal{M} = \mathcal{M}_{\gamma\text{-penguin}} + \mathcal{M}_{Z\text{-penguin}} + \mathcal{M}_{\text{box}}, \quad (3.5)$$

with

$$\begin{aligned} \mathcal{M}_{\gamma\text{-penguin}} = & \frac{e^2}{Q^2} \bar{u}(p_1) [Q^2 \gamma^\mu (A_1^L P_L + A_1^R P_R) + m_\mu i \sigma^{\mu\nu} Q_\nu (A_2^L P_L + A_2^R P_R)] u(p) \\ & \times \bar{u}(p_2) \gamma_\mu v(p_3) - (p_1 \leftrightarrow p_2), \end{aligned} \quad (3.6)$$

$$\begin{aligned} \mathcal{M}_{Z\text{-penguin}} = & \frac{e^2}{M_Z^2} \bar{u}(p_1) [\gamma^\mu (F_L P_L + F_R P_R)] u(p) \bar{u}(p_2) [\gamma_\mu (Z_L^e P_L + Z_R^e P_R)] v(p_3) \\ & - (p_1 \leftrightarrow p_2), \end{aligned} \quad (3.7)$$

$$\begin{aligned} \mathcal{M}_{\text{box}} = & e^2 B_1^L [\bar{u}(p_1) \gamma^\mu P_L u(p)] [\bar{u}(p_2) \gamma_\mu P_L v(p_3)] \\ & + e^2 B_1^R [\bar{u}(p_1) \gamma^\mu P_R u(p)] [\bar{u}(p_2) \gamma_\mu P_R v(p_3)] \\ & + e^2 B_2^L \{ [\bar{u}(p_1) \gamma^\mu P_L u(p)] [\bar{u}(p_2) \gamma_\mu P_R v(p_3)] - (p_1 \leftrightarrow p_2) \} \\ & + e^2 B_2^R \{ [\bar{u}(p_1) \gamma^\mu P_R u(p)] [\bar{u}(p_2) \gamma_\mu P_L v(p_3)] - (p_1 \leftrightarrow p_2) \} \\ & + e^2 B_3^L \{ [\bar{u}(p_1) P_L u(p)] [\bar{u}(p_2) P_L v(p_3)] - (p_1 \leftrightarrow p_2) \} \\ & + e^2 B_3^R \{ [\bar{u}(p_1) P_R u(p)] [\bar{u}(p_2) P_R v(p_3)] - (p_1 \leftrightarrow p_2) \} \\ & + e^2 B_4^L \{ [\bar{u}(p_1) \sigma^{\mu\nu} P_L u(p)] [\bar{u}(p_2) \sigma_{\mu\nu} P_L v(p_3)] - (p_1 \leftrightarrow p_2) \} \\ & + e^2 B_4^R \{ [\bar{u}(p_1) \sigma^{\mu\nu} P_R u(p)] [\bar{u}(p_2) \sigma_{\mu\nu} P_R v(p_3)] - (p_1 \leftrightarrow p_2) \}. \end{aligned} \quad (3.8)$$

We have defined new vertex form factors in the penguin amplitudes

$$\begin{aligned} A_1^L &= F_L^\gamma / Q^2, & A_1^R &= F_R^\gamma / Q^2, & A_2^L &= (F_M^\gamma + i F_E^\gamma) / m_\mu, & A_2^R &= (F_M^\gamma - i F_E^\gamma) / m_\mu, \\ F_L &= -F_L^Z, & F_R &= -F_R^Z, \end{aligned} \quad (3.9)$$

and used that $Q^2 \ll M_Z^2$ in eq. (3.7). $Z_{L,R}^e$ are the corresponding Z couplings to the electron in the SM (see appendix B). The dipole form factors $F_{M,E}^Z$ are dropped from the amplitude because their contributions are effectively suppressed by a factor m_μ^2 / M_{WH}^2 . The total width can then be written as [32, 35]:

$$\begin{aligned} \Gamma(\mu \rightarrow ee\bar{e}) = & \frac{\alpha^2 m_\mu^5}{32\pi} \left[|A_1^L|^2 + |A_1^R|^2 - 2(A_1^L A_2^{R*} + A_2^L A_1^{R*} + \text{h.c.}) \right. \\ & + (|A_2^L|^2 + |A_2^R|^2) \left(\frac{16}{3} \ln \frac{m_\mu}{m_e} - \frac{22}{3} \right) + \frac{1}{6} (|B_1^L|^2 + |B_1^R|^2) + \frac{1}{3} (|B_2^L|^2 + |B_2^R|^2) \\ & + \frac{1}{24} (|B_3^L|^2 + |B_3^R|^2) + 6(|B_4^L|^2 + |B_4^R|^2) - \frac{1}{2} (B_3^L B_4^{L*} + B_3^R B_4^{R*} + \text{h.c.}) \\ & \left. + \frac{1}{3} (A_1^L B_1^{L*} + A_1^R B_1^{R*} + A_1^L B_2^{L*} + A_1^R B_2^{R*} + \text{h.c.}) \right] \end{aligned}$$

$$\begin{aligned}
& -\frac{2}{3}(A_2^R B_1^{L*} + A_2^L B_1^{R*} + A_2^L B_2^{R*} + A_2^R B_2^{L*} + \text{h.c.}) \\
& + \frac{1}{3}\{2(|F_{LL}|^2 + |F_{RR}|^2) + |F_{LR}|^2 + |F_{RL}|^2 \\
& + (B_1^L F_{LL}^* + B_1^R F_{RR}^* + B_2^L F_{LR}^* + B_2^R F_{RL}^* + \text{h.c.}) + 2(A_1^L F_{LL}^* + A_1^R F_{RR}^* + \text{h.c.}) \\
& + (A_1^L F_{LR}^* + A_1^R F_{RL}^* + \text{h.c.}) - 4(A_2^R F_{LL}^* + A_2^L F_{RR}^* + \text{h.c.}) \\
& - 2(A_2^L F_{RL}^* + A_2^R F_{LR}^* + \text{h.c.})\}, \tag{3.10}
\end{aligned}$$

where

$$F_{LL} = \frac{F_L Z_L^e}{M_Z^2}, \quad F_{RR} = \frac{F_R Z_R^e}{M_Z^2}, \quad F_{LR} = \frac{F_L Z_R^e}{M_Z^2}, \quad F_{RL} = \frac{F_R Z_L^e}{M_Z^2}. \tag{3.11}$$

Note that the amplitude for the Z-penguin could have been cast into the box structure replacing

$$B_1^L \rightarrow B_1^L + 2F_{LL}, \tag{3.12}$$

$$B_1^R \rightarrow B_1^R + 2F_{RR}, \tag{3.13}$$

$$B_2^L \rightarrow B_2^L + F_{LR}, \tag{3.14}$$

$$B_2^R \rightarrow B_2^R + F_{RL}. \tag{3.15}$$

The branching ratios for both types of processes are obtained dividing by the SM decay width

$$\Gamma(\ell_j \rightarrow \ell_i \nu_j \bar{\nu}_i) = \frac{G_F^2 m_{\ell_j}^5}{192\pi^3}, \quad G_F = \frac{\pi\alpha}{\sqrt{2}s_W^2 M_W^2}. \tag{3.16}$$

For τ decays the SM branching ratio must be corrected multiplying by 0.17 to take into account other possible decay channels.

3.1 $\mu \rightarrow e\gamma$

Let us now summarize the calculation for $\mu \rightarrow e\gamma$. The new one-loop Feynman diagrams contributing to the $V\mu e$ vertex in the LHT model in the 't Hooft-Feynman gauge are listed in figure 3. They are classified in six topology classes. As explained above, in the decay $\mu \rightarrow e\gamma$ the photon is on-shell and then only the dipole form factors $F_{M,E}^\gamma$ contribute. They are proportional to the muon mass, reflecting the chirality flip character of the dipole transition. (The electron mass is neglected.) We separate the contributions exchanging W_H, Z_H and A_H , expressing the results in terms of standard loop integrals (appendix C).

Diagrams exchanging W_H . Taking $M_1 = M_{W_H}$ and $M_2 = m_{\nu_H^i}$ and introducing the mass ratio

$$y_i = \frac{m_{H^i}^2}{M_{W_H}^2}, \tag{3.17}$$

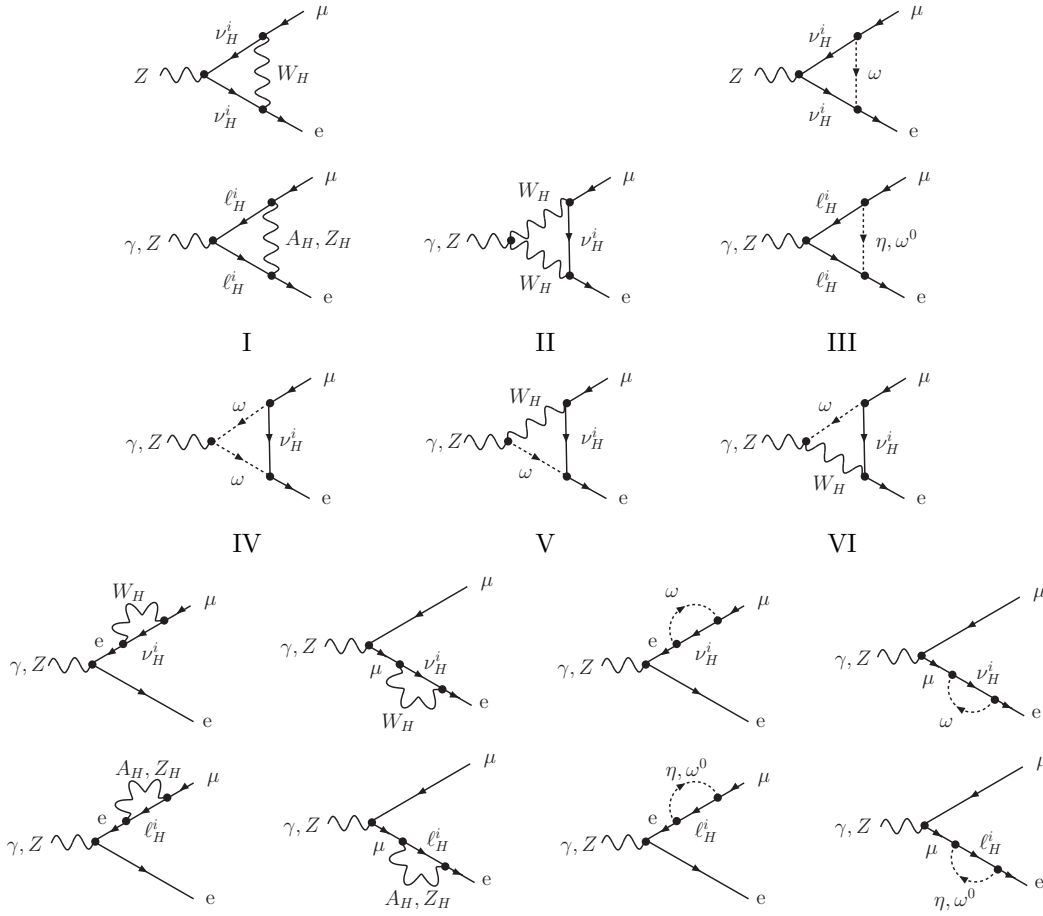


Figure 3: New one-loop diagrams contributing to $V_{\mu e}$ in the LHT model.

with $m_{Hi} \equiv m_{\ell_H^i} \simeq m_{\nu_H^i}$, we find the following contributions from diagrams exchanging W_H (see figure 3):

$$\text{II: } F_M^\gamma|_{W_H} = -iF_E^\gamma|_{W_H} = -\frac{\alpha_W}{16\pi} m_\mu \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} [3\bar{C}_{11} - \bar{C}_1], \quad (3.18)$$

$$\text{IV: } F_M^\gamma|_{W_H} = -iF_E^\gamma|_{W_H} = -\frac{\alpha_W}{16\pi} m_\mu \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} y_i \left[\bar{C}_0 + 3\bar{C}_1 + \frac{3}{2}\bar{C}_{11} \right], \quad (3.19)$$

$$\text{V: } F_M^\gamma|_{W_H} = -iF_E^\gamma|_{W_H} = 0, \quad (3.20)$$

$$\text{VI: } F_M^\gamma|_{W_H} = -iF_E^\gamma|_{W_H} = \frac{\alpha_W}{16\pi} m_\mu \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} \bar{C}_1, \quad (3.21)$$

$$\text{Total: } F_M^\gamma|_{W_H} = -iF_E^\gamma|_{W_H} = \frac{\alpha_W}{16\pi} \frac{m_\mu}{M_{W_H}^2} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} F_W(y_i), \quad (3.22)$$

where $\alpha_W \equiv \alpha/s_W^2$ and

$$\begin{aligned} F_W(x) &= M_1^2 \left[2\bar{C}_1 - 3\bar{C}_{11} - x \left(\bar{C}_0 + 3\bar{C}_1 + \frac{3}{2}\bar{C}_{11} \right) \right] \\ &= \frac{5}{6} - \frac{3x - 15x^2 - 6x^3}{12(1-x)^3} + \frac{3x^3}{2(1-x)^4} \ln x. \end{aligned} \quad (3.23)$$

The constant term drops from the amplitude due to the unitarity of the mixing matrix. This result is in agreement with [36, 37, 13].

It may be worth to note that these contributions are completely analogous to those of the SM with massive neutrinos, replacing W_H by W , ν_H^i by ν_i and $V_{H\ell}$ by V_{PMNS}^\dagger . For tiny neutrino masses, $x_i = m_{\nu_i}^2/M_W^2 \ll 1$,

$$F_W(x) \rightarrow \frac{5}{6} - \frac{x}{4} + \mathcal{O}(x^2), \quad (3.24)$$

and we recover a well known result [36] bounded by neutrino oscillation experiments:

$$\mathcal{B}(\mu \rightarrow e\gamma)_{\text{SM}} = \frac{3\alpha}{32\pi} \left| \sum_i V_{\text{PMNS}}^{ei} V_{\text{PMNS}}^{\mu i*} x_i \right|^2 \lesssim 10^{-54}. \quad (3.25)$$

Diagrams exchanging Z_H . Taking now $M_1 = M_{Z_H}$ and $M_2 = m_{\ell_H^i}$, with the same y_i , we get:

$$\text{I: } F_M^\gamma|_{Z_H} = -iF_E^\gamma|_{Z_H} = \frac{\alpha_W}{16\pi} m_\mu \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} \left[C_0 + 3C_1 + \frac{3}{2}C_{11} \right], \quad (3.26)$$

$$\text{III: } F_M^\gamma|_{Z_H} = -iF_E^\gamma|_{Z_H} = -\frac{\alpha_W}{32\pi} m_\mu \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} y_i \left[C_1 - \frac{3}{2}C_{11} \right], \quad (3.27)$$

$$\text{Total: } F_M^\gamma|_{Z_H} = -iF_E^\gamma|_{Z_H} = \frac{\alpha_W}{16\pi} \frac{m_\mu}{M_{W_H}^2} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} F_Z(y_i), \quad (3.28)$$

where

$$\begin{aligned} F_Z(x) &= M_1^2 \left[C_0 + 3C_1 + \frac{3}{2}C_{11} - \frac{x}{2} \left(C_1 - \frac{3}{2}C_{11} \right) \right] \\ &= -\frac{1}{3} + \frac{2x + 5x^2 - x^3}{8(1-x)^3} + \frac{3x^2}{4(1-x)^4} \ln x, \end{aligned} \quad (3.29)$$

in agreement with [37, 13].

Diagrams exchanging A_H . This contribution can be obtained from that of the diagrams with a Z_H , replacing Z_H by A_H . It is convenient to introduce the mass ratio

$$y'_i = ay_i, \quad a = \frac{M_{W_H}^2}{M_{A_H}^2} = \frac{5c_W^2}{s_W^2}. \quad (3.30)$$

Then,

$$\begin{aligned} F_M^\gamma|_{A_H} = -iF_E^\gamma|_{A_H} &= \frac{\alpha_W}{16\pi} \frac{m_\mu}{M_{A_H}^2} \frac{1}{25} \frac{s_W^2}{c_W^2} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} F_Z(y'_i) \\ &= \frac{\alpha_W}{16\pi} \frac{m_\mu}{M_{W_H}^2} \frac{1}{5} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} F_Z(y'_i). \end{aligned} \quad (3.31)$$

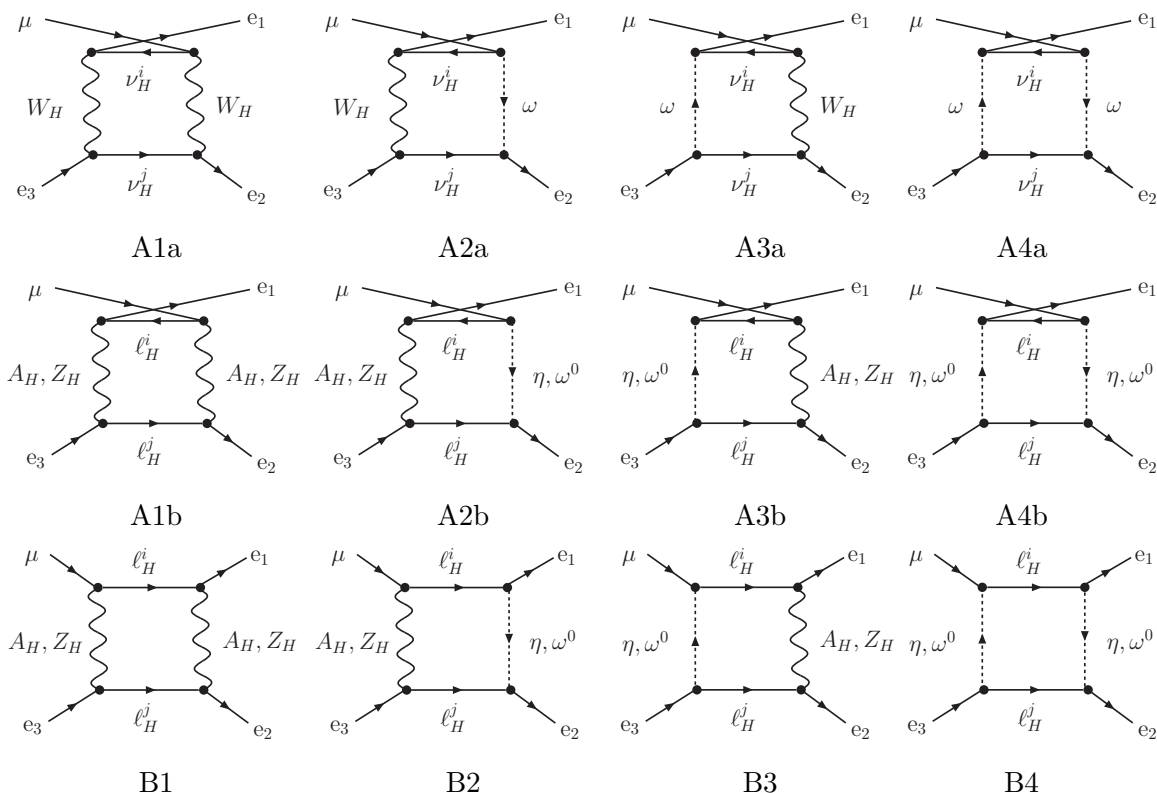


Figure 4: Box diagrams for $\mu \rightarrow e_1 e_2 \bar{e}_3$ in the LHT model. Crossed diagrams with e_1 and e_2 exchanged must be added.

Branching ratio. Using $M_W^2/M_{W_H}^2 = v^2/(4f^2)$ and $M_{W_H} = M_{Z_H}$, we finally obtain:

$$\mathcal{B}(\mu \rightarrow e\gamma) = \frac{3\alpha}{2\pi} \left| \frac{v^2}{4f^2} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} \left(F_W(y_i) + F_Z(y_i) + \frac{1}{5} F_Z(ay_i) \right) \right|^2, \quad (3.32)$$

with F_W and F_Z given in eqs. (3.23) and (3.29), respectively.

3.2 $\mu \rightarrow ee\bar{e}$

The self-energy diagrams do contribute to $F_{L,R}^V$ in this process, and must be included to calculate the penguin diagrams in $\mu \rightarrow ee\bar{e}$. On the other hand, apart from the box diagrams, only γ - and Z -penguin diagrams contribute to $\mu \rightarrow ee\bar{e}$ in the LHT model. This is so because A_H and Z_H do not couple to two ordinary fermions, as required by T-parity conservation. (Higgs-penguins vanish in the limit of massless electrons, as do F_R^V in this limit too.) For the sake of brevity we present our results grouping together the W_H , Z_H and A_H contributions, but we distinguish among the γ - and Z -penguins in figure 3 and the boxes in figure 4.

The γ -penguin. The form factors F_M^γ and F_E^γ have the same expressions (3.22), (3.28), (3.31) as for an on-shell photon, since terms of order Q^2 can

be neglected. The contributions to F_L^γ , which are proportional to $Q^2 \sim m_\mu^2$ as expected, are detailed below.

For $M_1 = M_{W_H}$, $M_2 = m_{\nu_H^i}$ and $y_i = m_{H^i}^2/M_{W_H}^2$ as before, the diagrams with W_H yield

$$\begin{aligned} F_L^\gamma|_{W_H} &= \frac{\alpha_W}{4\pi} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} G_W(y_i) \\ &= \frac{\alpha_W}{4\pi} \frac{Q^2}{M_{W_H}^2} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} G_W^{(1)}(y_i), \end{aligned} \quad (3.33)$$

with

$$\begin{aligned} G_W(x) &= -\frac{1}{2} + \bar{B}_1 + 6\bar{C}_{00} + x \left(\frac{1}{2}\bar{B}_1 + \bar{C}_{00} - M_1^2\bar{C}_0 \right) - \left(2\bar{C}_1 + \frac{1}{2}\bar{C}_{11} \right) Q^2 \\ &= \Delta_\epsilon - \ln \frac{M_1^2}{\mu^2} + \frac{Q^2}{M_1^2} G_W^{(1)}(x) + \mathcal{O}\left(\frac{Q^4}{M_1^4}\right), \end{aligned} \quad (3.34)$$

$$G_W^{(1)}(x) = -\frac{5}{18} + \frac{x(12+x-7x^2)}{24(1-x)^3} + \frac{x^2(12-10x+x^2)}{12(1-x)^4} \ln x. \quad (3.35)$$

Relations (C.20) and (C.22) have been used in (3.34). Note that owing to the unitarity of the mixing matrix the x -independent terms in $G_W(x)$ drop out (including the ultraviolet divergence). The SM prediction is obtained by replacing W_H by W , ν_H^i by ν_i and $V_{H\ell}$ by V_{PMNS}^\dagger .

For $M_1 = M_{Z_H}(= M_{W_H})$, $M_2 = m_{\ell_H^i} \simeq m_{\nu_H^i}$ and the same y_i , the contribution of diagrams with Z_H is

$$F_L^\gamma|_{Z_H} = \frac{\alpha_W}{4\pi} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} G_Z(y_i) \quad (3.36)$$

$$= \frac{\alpha_W}{4\pi} \frac{Q^2}{M_{W_H}^2} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} G_Z^{(1)}(y_i), \quad (3.37)$$

with

$$\begin{aligned} G_Z(x) &= \left(1 + \frac{x}{2}\right) \left(-\frac{1}{4} + \frac{1}{2}\bar{B}_1 + C_{00} - \frac{x}{2}M_1^2C_0\right) - \left(\frac{1}{2}C_0 + C_1 + \frac{1}{8}(2+x)C_{11}\right) Q^2 \\ &= \frac{Q^2}{M_1^2} G_Z^{(1)}(x) + \mathcal{O}\left(\frac{Q^4}{M_1^4}\right), \end{aligned} \quad (3.38)$$

$$G_Z^{(1)}(x) = \frac{1}{36} + \frac{x(18-11x-x^2)}{48(1-x)^3} - \frac{4-16x+9x^2}{24(1-x)^4} \ln x. \quad (3.39)$$

The relation (C.22) has been used in eq. (3.38).

Finally, the contribution of diagrams with A_H is obtained from that of diagrams with Z_H replacing Z_H by A_H , and y_i by $y'_i = 5c_W^2 y_i/s_W^2$:

$$F_L^\gamma|_{A_H} = \frac{\alpha_W}{4\pi} \frac{Q^2}{M_{W_H}^2} \frac{1}{5} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} G_Z^{(1)}(y'_i). \quad (3.40)$$

The Z-penguin. The Z dipole form factors $F_{M,E}^Z$ (which are chirality flipping and hence proportional to the muon mass) can be neglected as compared to F_L^Z . This is in contrast with the γ -penguin, for which $QF_{M,E}^\gamma (\sim QF_{M,E}^Z) \sim Q^2/M_{W_H}^2 \lesssim m_\mu^2/M_{W_H}^2 \sim F_L^\gamma$, to be compared with $F_L^Z \sim 1$. This justifies to neglect $F_{M,E}^Z$ in the Z-penguin (3.7).

Taking $M_1 = M_{W_H}$, $M_2 = m_{H_i}$ and $y_i = m_{H_i}^2/M_{W_H}^2$, and using the unitarity of $V_{H\ell}$ we obtain:

$$\begin{aligned}
 F_L^Z|_{W_H} &= \frac{\alpha_W}{8\pi} \frac{1}{s_W c_W} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} \left\{ -2c_W^2 \left(-\frac{1}{2} + \bar{B}_1 + 6\bar{C}_{00} - y_i M_{W_H}^2 \bar{C}_0 \right) \right. \\
 &\quad \left. - y_i c_W^2 (\bar{B}_1 + 2\bar{C}_{00}) \right. \\
 &\quad \left. + 2 \left(1 + \frac{y_i}{2} \right) \left(-\frac{1}{4} + \frac{1}{2} \bar{B}_1 + C_{00} - \frac{y_i}{2} M_{W_H}^2 C_0 \right) \right. \\
 &\quad \left. + \frac{v^2}{f^2} \frac{y_i}{16} [1 + 4(\bar{C}_{00} - C_{00} + M_{W_H}^2 (C_0 - 2\bar{C}_0))] \right\} \\
 &= \frac{\alpha_W}{8\pi} \frac{1}{s_W c_W} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} \left\{ -2c_W^2 \left(\Delta_\epsilon - \ln \frac{M_{W_H}^2}{\mu^2} \right) + \frac{v^2}{f^2} \frac{y_i}{8} H_W(y_i) \right\} \\
 &= \frac{\alpha_W}{8\pi} \frac{1}{s_W c_W} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} \frac{v^2}{f^2} \frac{y_i}{8} H_W(y_i), \tag{3.41}
 \end{aligned}$$

with

$$H_W(x) = \frac{6-x}{1-x} + \frac{2+3x}{(1-x)^2} \ln x, \tag{3.42}$$

and

$$\begin{aligned}
 F_L^Z|_{Z_H} &= \frac{\alpha_W}{8\pi} \frac{1}{s_W c_W} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} (1 - 2c_W^2) \left(-\frac{1}{4} + \frac{1}{2} \bar{B}_1 + C_{00} - \frac{y_i}{2} M_{W_H}^2 C_0 \right) \\
 &\quad \times \left\{ \left(1 + \frac{y_i}{2} \right) - \frac{v^2}{f^2} \left[\frac{y_i}{4} + \left(\frac{c_W}{s_W} y_i - \frac{2s_W}{5c_W} \right) x_H \right] \right\} = 0, \tag{3.43}
 \end{aligned}$$

$$\begin{aligned}
 F_L^Z|_{A_H} &= \frac{\alpha_W}{8\pi} \frac{1}{s_W c_W} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} (1 - 2c_W^2) \left(-\frac{1}{4} + \frac{1}{2} \bar{B}_1 + C_{00} - \frac{y_i}{2} M_{A_H}^2 C_0 \right) \\
 &\quad \times \frac{1}{25} \frac{s_W^2}{c_W^2} \left\{ \left(1 + \frac{y_i'}{2} \right) - \frac{v^2}{f^2} \left[\frac{5}{4} y_i' + \left(\frac{s_W}{c_W} y_i' + 10 \frac{c_W}{s_W} \right) x_H \right] \right\} = 0. \tag{3.44}
 \end{aligned}$$

Here x_H is a constant defining the mixing between the heavy neutral gauge bosons and function of the gauge couplings (see eq. (A.4)). We observe that the only contribution to the Z-penguins comes from the diagrams with W_H , and it is proportional to v^2/f^2 . The potentially dangerous ultraviolet divergences proportional to y_i have cancelled thanks to the proper v^2/f^2 corrections to the $\omega^\pm W_H^\mp Z$ and $Z \bar{\nu}_{HR}^i \nu_{HR}^i$ couplings. The corrections to the latter were not included in [13].

For completeness, we give the prediction for the Z-penguin in the SM with light massive neutrinos. Although in the LHT the heavy leptons are vector-like and the Z boson couples to both chiralities, the final form of the vertex is the same. This is more easily seen in the unitary gauge, where the heavy modes contribution is only given by diagram I in figure 3

and is proportional to the v^2/f^2 correction to the $Z\bar{\nu}_{HR}^i\nu_{HR}^i$ coupling (see [26] for further discussion). Taking $M_1 = M_W$, $M_2 = m_{\nu_i}$ and $x_i = m_{\nu_i}^2/M_W^2$, and using the unitarity of V_{PMNS} we obtain:

$$\begin{aligned}
F_L^Z|_W &= \frac{\alpha_W}{8\pi} \frac{1}{s_W c_W} \sum_i V_{\text{PMNS}}^{ei} V_{\text{PMNS}}^{\mu i*} \left\{ -2c_W^2 \left(-\frac{1}{2} + \bar{B}_1 + 6\bar{C}_{00} - x_i M_W^2 \bar{C}_0 \right) \right. \\
&\quad \left. + \frac{x_i}{2} (1 - 2c_W^2) (\bar{B}_1 + 2\bar{C}_{00}) \right. \\
&\quad \left. - \frac{1}{2} + \bar{B}_1 + 2C_{00} - \frac{x_i}{2} M_W^2 (4\bar{C}_0 + x_i C_0) \right\} \\
&= \frac{\alpha_W}{16\pi} \frac{1}{s_W c_W} \sum_i V_{\text{PMNS}}^{ei} V_{\text{PMNS}}^{\mu i*} \left\{ -4c_W^2 \left(\Delta_\epsilon - \ln \frac{M_W^2}{\mu^2} \right) - 1 \right. \\
&\quad \left. + 2(\bar{B}_1 + 2C_{00}) - x_i M_W^2 (4\bar{C}_0 + x_i C_0) \right\} \\
&= \frac{\alpha_W}{16\pi} \frac{1}{s_W c_W} \sum_i V_{\text{PMNS}}^{ei} V_{\text{PMNS}}^{\mu i*} x_i H_W(x_i), \tag{3.45}
\end{aligned}$$

which is, of course, finite and in agreement with ref. [33] for $Q^2 = 0$.

Box diagrams. There are eight different classes of box diagrams grouped in types A and B in the LHT model (figure 4). In the limit of zero external momenta (all internal masses are much larger than the muon mass) all of them have the same form, being proportional to a scalar integral over the internal momentum q . Indeed, omitting the corresponding denominator $(q^2 - m_{Hi}^2)^2(q^2 - M_{G_H}^2)^2$, with $G = W, Z$ or A ,

$$\begin{aligned}
\text{A1} : & \langle p_1 | \gamma^\mu P_L (-\not{q} + m_{Hi}) \gamma^\nu P_L | p \rangle \langle p_2 | \gamma_\nu P_L (-\not{q} + m_{Hi}) \gamma_\mu P_L | p_3 \rangle \\
&= \frac{q^2}{4} \langle p_1 | \gamma^\mu \gamma^\alpha \gamma^\nu P_L | p \rangle \langle p_2 | \gamma_\nu \gamma_\alpha \gamma_\mu P_L | p_3 \rangle \\
&= q^2 \langle p_1 | \gamma^\mu P_L | p \rangle \langle p_2 | \gamma_\mu P_L | p_3 \rangle, \tag{3.46}
\end{aligned}$$

$$\begin{aligned}
\text{A2} : & - \langle p_1 | \gamma^\mu P_L (-\not{q} + m_{Hi}) P_L | p \rangle \langle p_2 | P_R (-\not{q} + m_{Hi}) \gamma_\mu P_L | p_3 \rangle \\
&= -m_{Hi} m_{Hj} \langle p_1 | \gamma^\mu P_L | p \rangle \langle p_2 | \gamma_\mu P_L | p_3 \rangle, \tag{3.47}
\end{aligned}$$

$$\begin{aligned}
\text{A3} : & - \langle p_1 | P_R (-\not{q} + m_{Hi}) \gamma^\mu P_L | p \rangle \langle p_2 | \gamma_\mu P_L (-\not{q} + m_{Hi}) P_L | p_3 \rangle \\
&= -m_{Hi} m_{Hj} \langle p_1 | \gamma^\mu P_L | p \rangle \langle p_2 | \gamma_\mu P_L | p_3 \rangle, \tag{3.48}
\end{aligned}$$

$$\begin{aligned}
\text{A4} : & \langle p_1 | P_R (-\not{q} + m_{Hi}) P_L | p \rangle \langle p_2 | P_R (-\not{q} + m_{Hi}) P_L | p_3 \rangle \\
&= \frac{q^2}{4} \langle p_1 | \gamma^\mu P_L | p \rangle \langle p_2 | \gamma_\mu P_L | p_3 \rangle, \tag{3.49}
\end{aligned}$$

$$\begin{aligned}
\text{B1} : & \langle p_1 | \gamma^\mu P_L (\not{q} + m_{Hi}) \gamma^\nu P_L | p \rangle \langle p_2 | \gamma_\nu P_L (-\not{q} + m_{Hi}) \gamma_\mu P_L | p_3 \rangle \\
&= -\frac{q^2}{4} \langle p_1 | \gamma^\mu \gamma^\alpha \gamma^\nu P_L | p \rangle \langle p_2 | \gamma_\mu \gamma_\alpha \gamma_\nu P_L | p_3 \rangle \\
&= -4q^2 \langle p_1 | \gamma^\mu P_L | p \rangle \langle p_2 | \gamma_\mu P_L | p_3 \rangle, \tag{3.50}
\end{aligned}$$

$$\begin{aligned}
\text{B2} : & - \langle p_1 | \gamma^\mu P_L (\not{q} + m_{Hi}) P_L | p \rangle \langle p_2 | P_R (-\not{q} + m_{Hi}) \gamma_\mu P_L | p_3 \rangle \\
&= -m_{Hi} m_{Hj} \langle p_1 | \gamma^\mu P_L | p \rangle \langle p_2 | \gamma_\mu P_L | p_3 \rangle, \tag{3.51}
\end{aligned}$$

$$\begin{aligned}
 \text{B3} : & - \langle p_1 | P_R(\not{q} + m_{Hi})\gamma^\mu P_L | p \rangle \langle p_2 | \gamma_\mu P_L(-\not{q} + m_{Hi})P_L | p_3 \rangle \\
 & = -m_{Hi}m_{Hj} \langle p_1 | \gamma^\mu P_L | p \rangle \langle p_2 | \gamma_\mu P_L | p_3 \rangle, \tag{3.52}
 \end{aligned}$$

$$\begin{aligned}
 \text{B4} : & \langle p_1 | P_R(\not{q} + m_{Hi})P_L | p \rangle \langle p_2 | P_R(-\not{q} + m_{Hi})P_L | p_3 \rangle \\
 & = -\frac{q^2}{4} \langle p_1 | \gamma^\mu P_L | p \rangle \langle p_2 | \gamma_\mu P_L | p_3 \rangle. \tag{3.53}
 \end{aligned}$$

Thus, all box form factors except B_1^L vanish (see eq. (3.8)). Using the Fierz identity

$$\langle 1 | \gamma^\mu P_L | 2 \rangle \langle 3 | \gamma_\mu P_L | 4 \rangle = - \langle 3 | \gamma^\mu P_L | 2 \rangle \langle 1 | \gamma_\mu P_L | 4 \rangle \tag{3.54}$$

and including all the factors, we obtain the generic expressions for the contributions from diagrams of types A and B (see figure 4 and appendix B for definitions),

$$\begin{aligned}
 \text{A} : \quad B_1^L & = 2\frac{\alpha}{4\pi} \sum_{ij} \left[\left(g_{L1}^{ie*} g_{L2}^{i\mu} g_{L1}^{je} g_{L2}^{je*} + \frac{1}{4} c_{L1}^{ie*} c_{L2}^{i\mu} c_{L1}^{je} c_{L2}^{je*} \right) \tilde{D}_0(M_1^2, M_2^2, m_{Hi}^2, m_{Hj}^2) \right. \\
 & \quad \left. - \left(g_{L1}^{ie*} c_{L2}^{i\mu} g_{L1}^{je} c_{L2}^{je*} + c_{L1}^{ie*} g_{L2}^{i\mu} c_{L1}^{je} g_{L2}^{je*} \right) m_{Hi} m_{Hj} D_0(M_1^2, M_2^2, m_{Hi}^2, m_{Hj}^2) \right], \tag{3.55}
 \end{aligned}$$

$$\begin{aligned}
 \text{B} : \quad B_1^L & = 2\frac{\alpha}{4\pi} \sum_{ij} \left[- \left(4g_{L2}^{ie*} g_{L1}^{i\mu} g_{L1}^{je} g_{L2}^{je*} + \frac{1}{4} c_{L2}^{ie*} c_{L1}^{i\mu} c_{L1}^{je} c_{L2}^{je*} \right) \tilde{D}_0(M_1^2, M_2^2, m_{Hi}^2, m_{Hj}^2) \right. \\
 & \quad \left. - \left(g_{L2}^{ie*} c_{L1}^{i\mu} c_{L1}^{je} g_{L2}^{je*} + c_{L2}^{ie*} g_{L1}^{i\mu} g_{L1}^{je} c_{L2}^{je*} \right) m_{Hi} m_{Hj} D_0(M_1^2, M_2^2, m_{Hi}^2, m_{Hj}^2) \right]. \tag{3.56}
 \end{aligned}$$

Finally, replacing the vertex coefficients given in appendix B we derive the contributions of the heavy gauge bosons and the corresponding would-be-Goldstone bosons:

$$B_1^L(W_H, W_H) = \frac{\alpha}{2\pi} \frac{1}{4s_W^4} \frac{1}{M_W^2} \frac{v^2}{4f^2} \sum_{ij} \chi_{ij} \left[\left(1 + \frac{1}{4} y_i y_j \right) \tilde{d}_0(y_i, y_j) - 2y_i y_j d_0(y_i, y_j) \right], \tag{3.57}$$

$$B_1^L(Z_H, Z_H) = \frac{\alpha}{2\pi} \frac{1}{16s_W^4} \frac{1}{M_W^2} \frac{v^2}{4f^2} \sum_{ij} \chi_{ij} \left[-3\tilde{d}_0(y_i, y_j) \right], \tag{3.58}$$

$$B_1^L(A_H, A_H) = \frac{\alpha}{2\pi} \frac{1}{16s_W^4} \frac{1}{25a} \frac{1}{M_W^2} \frac{v^2}{4f^2} \sum_{ij} \chi_{ij} \left[-3\tilde{d}_0(y'_i, y'_j) \right], \tag{3.59}$$

$$B_1^L(Z_H, A_H) = \frac{\alpha}{2\pi} \frac{1}{16s_W^4} \frac{2}{5} \frac{1}{M_W^2} \frac{v^2}{4f^2} \sum_{ij} \chi_{ij} \left[-3\tilde{d}_0(a, y'_i, y'_j) \right], \tag{3.60}$$

with

$$\chi_{ij} = V_{H\ell}^{ie*} V_{H\ell}^{i\mu} |V_{H\ell}^{je}|^2. \tag{3.61}$$

The SM contribution from the exchange of the light neutrinos is again similar to that from W_H , but performing the corresponding replacements.

Branching ratio. Collecting everything, the non-vanishing contributions to the vertex and box form factors in eq. (3.10) from γ -penguins, Z-penguins and box diagrams in the LHT can be written

$$A_1^L = \frac{F_L^\gamma}{Q^2} = \frac{\alpha_W}{4\pi} \frac{1}{M_W^2} \frac{v^2}{4f^2} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} \left[G_W^{(1)}(y_i) + G_Z^{(1)}(y_i) + \frac{1}{5} G_Z^{(1)}(ay_i) \right], \tag{3.62}$$

$$A_2^R = \frac{2F_M^\gamma}{m_\mu} = \frac{\alpha_W}{8\pi} \frac{1}{M_W^2} \frac{v^2}{4f^2} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} \left[F_W(y_i) + F_Z(y_i) + \frac{1}{5} F_Z(ay_i) \right], \quad (3.63)$$

$$F_{LL} = -\frac{F_L^Z Z_L^e}{M_Z^2} = \frac{\alpha_W}{8\pi} \frac{1 - 2s_W^2}{2s_W^2} \frac{1}{M_W^2} \frac{v^2}{f^2} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} \frac{y_i}{8} H_W(y_i), \quad (3.64)$$

$$F_{LR} = -\frac{F_L^Z Z_R^e}{M_Z^2} = -\frac{\alpha_W}{8\pi} \frac{1}{M_W^2} \frac{v^2}{f^2} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} \frac{y_i}{8} H_W(y_i), \quad (3.65)$$

$$B_1^L = \frac{\alpha_W}{8\pi} \frac{1}{s_W^2} \frac{1}{M_W^2} \frac{v^2}{4f^2} \sum_{ij} \chi_{ij} \left[\left(1 + \frac{1}{4} y_i y_j \right) \tilde{d}_0(y_i, y_j) - 2y_i y_j d_0(y_i, y_j) - \frac{3}{4} \tilde{d}_0(y_i, y_j) - \frac{3}{100a} \tilde{d}_0(ay_i, ay_j) - \frac{3}{10} \tilde{d}_0(a, ay_i, ay_j) \right]. \quad (3.66)$$

The branching ratio reads

$$\begin{aligned} \mathcal{B}(\mu \rightarrow ee\bar{e}) = & 12s_W^4 M_W^4 \left\{ |A_1^L|^2 - 2(A_1^L A_2^{R*} + \text{h.c.}) + |A_2^R|^2 \left(\frac{16}{3} \ln \frac{m_\mu}{m_e} - \frac{22}{3} \right) \right. \\ & + \frac{1}{6} |B_1^L|^2 + \frac{1}{3} (A_1^L B_1^{L*} + \text{h.c.}) - \frac{2}{3} (A_2^R B_1^{L*} + \text{h.c.}) + \frac{1}{3} (2|F_{LL}|^2 + |F_{LR}|^2) \\ & \left. + \frac{1}{3} (B_1^L F_{LL}^* + 2A_1^L F_{LL}^* + A_1^L F_{LR}^* - 4A_2^R F_{LL}^* - 2A_2^R F_{LR}^* + \text{h.c.}) \right\}. \quad (3.67) \end{aligned}$$

4. Numerical results

In order to study the bounds on the new parameters imposed by the experimental limits on $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee\bar{e}$, it is convenient to restrict ourselves to the case of two generations. Hence, we are left with four parameters: the LH order parameter f , the masses of the two heavy lepton doublets in (A.18) m_{H_i} ($i = 1, 2$), and the angle θ defining the 2×2 mixing matrix between the heavy and the SM charged leptons

$$V_{H\ell} = \begin{pmatrix} V_{H\ell}^{1e} & V_{H\ell}^{1\mu} \\ V_{H\ell}^{2e} & V_{H\ell}^{2\mu} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (4.1)$$

(In the contributions we consider the e and μ phases, as well as the heavy lepton doublet phases, can be safely redefined.) We shall replace m_{H_1} , m_{H_2} by δ and \tilde{y} , however, to present our results. The former, which is proportional to the heavy lepton mass difference, describes together with θ the alignment between heavy and SM charged leptons,

$$\delta = \frac{m_{H_2}^2 - m_{H_1}^2}{m_{H_1} m_{H_2}}. \quad (4.2)$$

Whereas the latter, which sets the heavy lepton scale, is relevant for discussing decoupling,

$$\tilde{y} = \sqrt{y_1 y_2}, \quad y_i = \frac{m_{H_i}^2}{M_{W_H}^2}, \quad i = 1, 2. \quad (4.3)$$

Note that both M_{W_H} and m_{H_i} are proportional to f (see appendix A). The penguin contributions then take the form

$$\sum_{i=1}^2 V_{H\ell}^{ie*} V_{H\ell}^{i\mu} F(y_i) = \frac{\sin 2\theta}{2} [F(y_1) - F(y_2)], \quad (4.4)$$

where F stands for a generic function; and the box contributions

$$\sum_{i,j=1}^2 V_{H\ell}^{ie*} V_{H\ell}^{i\mu} |V_{H\ell}^{je}|^2 F(y_i, y_j) = \frac{\sin 2\theta}{2} \left\{ \cos^2 \theta [F(y_1, y_1) - F(y_2, y_1)] + \sin^2 \theta [F(y_1, y_2) - F(y_2, y_2)] \right\}. \quad (4.5)$$

Thus, the LFV amplitudes vanish for vanishing mixing, $\theta = 0$, or heavy mass splitting, $\delta = 0$.

We plot for illustration in figure 5 the form factors for the $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee\bar{e}$ decay amplitudes calculated in the previous section as a function of δ for several θ and \tilde{y} values and $f = 1$ TeV. They grow with \tilde{y} and scale like f^{-2} . In contrast with the MSSM case [32, 35], box contributions to $\mu \rightarrow ee\bar{e}$ are of the same order than penguins, in particular for $\tilde{y} \gtrsim 1$, which explains the different behaviour of the decay rates with the sign of δ for non-maximal flavour mixing. The dependence on the new parameters is more clearly seen in figures 6 and 7. They show the present exclusion contours in the $(\sin 2\theta, \delta)$ plane implied by the present limits on $\mathcal{B}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ [38] and $\mathcal{B}(\mu \rightarrow ee\bar{e}) < 10^{-12}$ [39], respectively, and for three values of \tilde{y} , 0.25, 1, 4. The regions above each line of constant f are excluded. As it can be observed, mixing angle and mass splitting are correlated, because the alignment between the Yukawa couplings of the heavy and the SM charged leptons goes to zero with any of them. Present limits on LFV muon decays imply that θ or $\delta \lesssim 0.1$ for $\tilde{y} = 1$ and $f = 1$ TeV. If no LFV signal is seen by the MEG experiment at PSI, the limits are expected to improve by two orders of magnitude [40] and the corresponding exclusion contours would be those in figures 6 and 7 replacing f by $\sqrt{10}f$.

As already emphasized, the LFV branching ratios scale like f^{-4} . However, the \tilde{y} dependence deserves more discussion. In figure 8 we plot the variation of the form factors and of the branching ratios with \tilde{y} for maximal mixing, $\sin 2\theta = 1$, and $\delta = 1$. Two comments are in order. The non-observation of these LFV processes already sets non-trivial limits on the LHT parameters because the central region $\tilde{y} \sim 1$ is already excluded for natural values of the other parameters. More interestingly, $\mathcal{B}(\mu \rightarrow ee\bar{e})$ goes like \tilde{y}^2 for very large \tilde{y} . This is so because \tilde{y} is quadratic in the heavy Yukawa coupling κ , which goes to infinity with the heavy lepton masses for fixed f . This behaviour is similar to the leading EWPD dependence on the top quark mass [41], which scales with m_t^2 in the region of physical interest, allowing a determination of the top mass from a global fit [42]. Just like in the top quark case, the dependence is moderate when the particles within multiplets become degenerate (the symmetry is recovered). Generic limits from all these figures are tabulated in the summary below.

Let us, finally, comment on the general case with three families. Similarly to the two family case, we have to align the new contributions to the electron and to the muon at the 10% level. However, now this alignment is not easily related to the usual parameterization

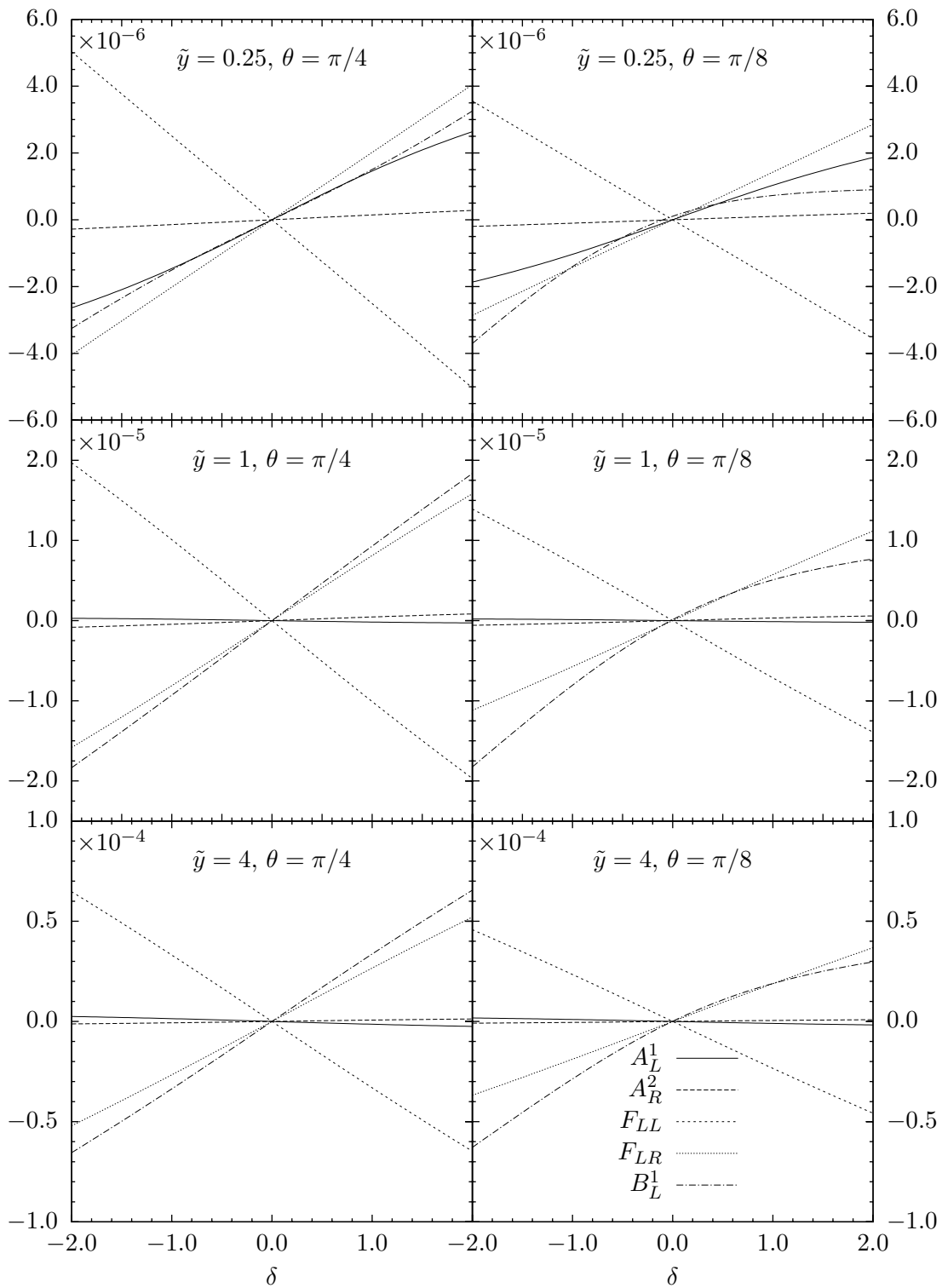


Figure 5: Form factors multiplied by M_W^2 , for $f = 1$ TeV.

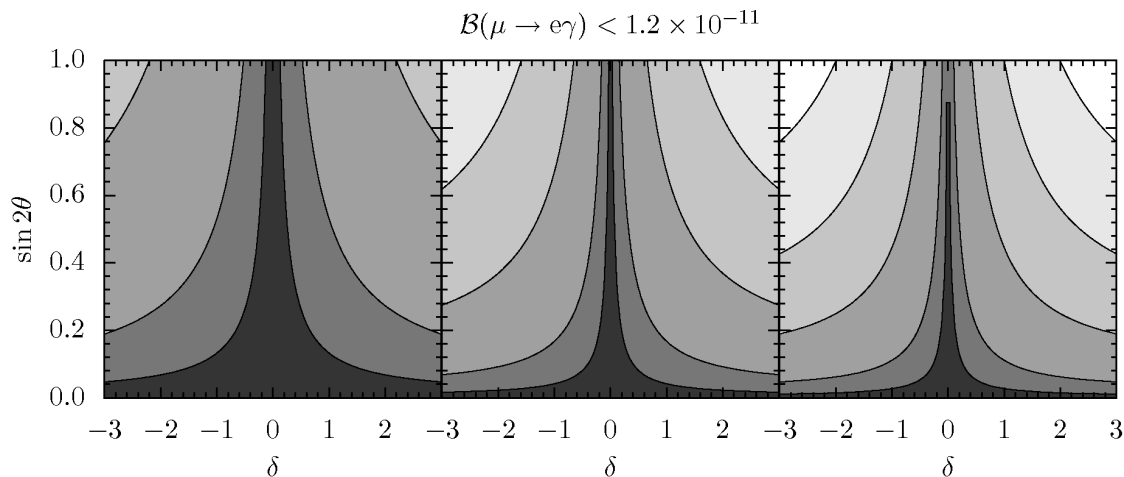


Figure 6: Contours of $\mathcal{B}(\mu \rightarrow e\gamma) = 1.2 \times 10^{-11}$ in the $(\sin 2\theta, \delta)$ plane for $\tilde{y} = 0.25, 1, 4$ (left, center, right) and $f = 0.5, 1, 2, 3, 4$ TeV (from bottom up).

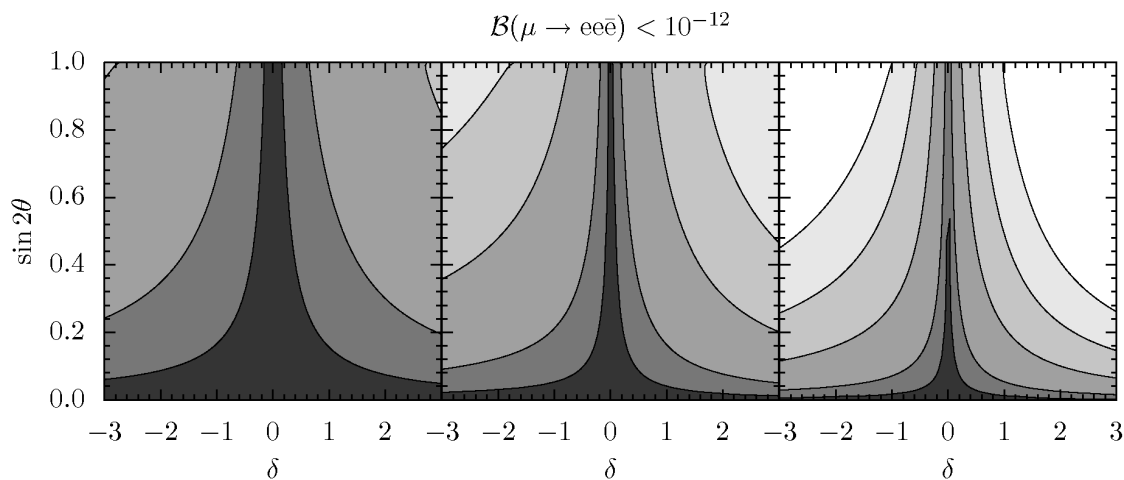


Figure 7: Contours of $\mathcal{B}(\mu \rightarrow ee\bar{e}) = 10^{-12}$ in the $(\sin 2\theta, \delta)$ plane for $\tilde{y} = 0.25, 1, 4$ (left, center, right) and $f = 0.5, 1, 2, 3, 4$ TeV (from bottom up).

of the mixing in terms of two mass splittings, three mixing angles and one phase (as in the two family case we can safely redefine the e and μ , as well as the heavy lepton doublet phases, and then use the same parameterization as for the CKM matrix [42]). In order to estimate the fine tuning required by $\mu \rightarrow e\gamma$, for instance, we rather introduce the ratio (see eq. (4.4))

$$\frac{\left| \sum_{i=1}^3 V_{H\ell}^{ie*} V_{H\ell}^{i\mu} F(y_i) \right|^2}{\left(\sum_{i=1}^3 |V_{H\ell}^{ie}|^2 |F(y_i)| \right) \left(\sum_{i=1}^3 |V_{H\ell}^{i\mu}|^2 |F(y_i)| \right)}, \quad (4.6)$$

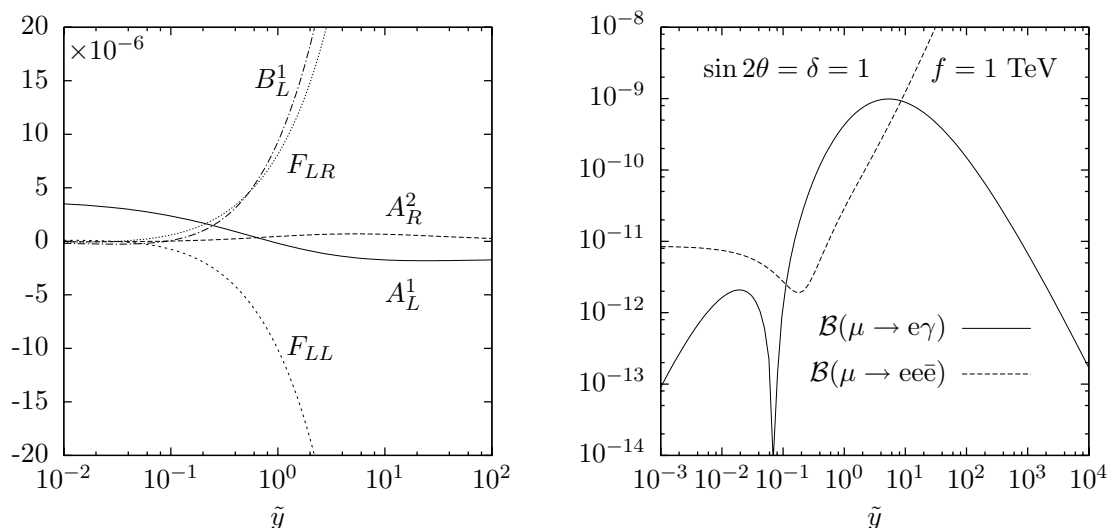


Figure 8: Form factors multiplied by M_W^2 (left) and branching ratios (right) as a function of \tilde{y} for $\sin 2\theta = \delta = 1$ and $f = 1 \text{ TeV}$. The latter must be compared with present experimental limits on $\mathcal{B}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ and $\mathcal{B}(\mu \rightarrow ee\bar{e}) < 10^{-12}$.

which approximately scales like $\sin^2 2\theta\delta^2$ for two families. This is $\lesssim 10^{-2}$ for almost all the three family parameter space when $\mathcal{B}(\mu \rightarrow e\gamma)$ is below the present experimental limit, and at most $\sim 10^{-4}$ if the limit improves by two orders of magnitude. There is a special region in parameter space, however, where the ratio (4.6) can be larger even though $\mathcal{B}(\mu \rightarrow e\gamma)$ is well below the experimental limit. This is around $y_i \sim 0.3$, where the total amplitude $F(y_i) \sim F_W(y_i) + F_Z(y_i) + \frac{1}{5}F_Z(ay_i)$ in eq. (3.32) is negligible (see right panel of figure 8). But this region is excluded by the present limit on $\mathcal{B}(\mu \rightarrow ee\bar{e})$. Thus, the new contributions to the electron and to the muon must be aligned at the 10% level, being the square of this precision the largest value of the ratio in eq. (4.6). Analogously, we can define the corresponding ratio using the amplitudes for $\mu \rightarrow ee\bar{e}$ in eqs. (4.4), (4.5), obtaining similar results. The numerical analysis presented here is at some extent complementary to the study in ref. [13], where the correlation between different observables, in particular between $\mathcal{B}(\mu \rightarrow e\gamma)$ and $\mathcal{B}(\mu \rightarrow ee\bar{e})$, is explicitly shown.

5. Conclusions

LH models provide a natural explanation of the little hierarchy between the EW scale and the scale where we expect the NP to be, and which is to be explored by the LHC. However, these models where the Higgs is a pseudo-Goldstone boson of an approximate global symmetry in general suffer some tension in accommodating the many new particles required near the TeV scale without upsetting the EWPD constraints. This is ameliorated by further extending the model to include a discrete symmetry, the T parity, under which all observed particles, including the Higgs boson, are even and hopefully all the new ones are odd. All these models, as any *universal* NP near the TeV scale, must also guarantee

$\mathcal{B}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11} \quad (\sin 2\theta = 1)$			
f [TeV]	$\tilde{y} = 0.25$	$\tilde{y} = 1$	$\tilde{y} = 4$
0.5	$ \delta < 0.13$	$ \delta < 0.040$	$ \delta < 0.027$
1.0	$ \delta < 0.52$	$ \delta < 0.16$	$ \delta < 0.11$
2.0	$ \delta < 2.2$	$ \delta < 0.66$	$ \delta < 0.43$
4.0	$ \delta < 14$	$ \delta < 3.5$	$ \delta < 2.0$

Table 1: Bounds on the splitting δ from the present experimental limit on $\mathcal{B}(\mu \rightarrow e\gamma)$ for several scales f and ratios \tilde{y} , taking $\sin 2\theta = 1$.

$\mathcal{B}(\mu \rightarrow ee\bar{e}) < 10^{-12} \quad (\sin 2\theta = 1)$			
f [TeV]	$\tilde{y} = 0.25$	$\tilde{y} = 1$	$\tilde{y} = 4$
0.5	$ \delta < 0.16$	$ \delta < 0.045$	$ \delta < 0.015$
1.0	$ \delta < 0.64$	$ \delta < 0.18$	$ \delta < 0.061$
2.0	$ \delta < 2.7$	$ \delta < 0.72$	$ \delta < 0.24$
4.0	$ \delta < 13$	$ \delta < 3.3$	$ \delta < 0.98$

Table 2: Bounds on the splitting δ from the present experimental limits on $\mathcal{B}(\mu \rightarrow ee\bar{e})$ for several scales f and ratios \tilde{y} , taking $\sin 2\theta = 1$.

that the new particles do not mediate too large FCNC processes. We have recalculated the new contributions to the LFV processes $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee\bar{e}$ in the LHT model, the most economical of such proposals. The full Lagrangian has been introduced and all pieces of the calculation, in particular the Z-penguin and box diagrams contributing to $\mu \rightarrow ee\bar{e}$, have been considered in detail. We have found that the former are ultraviolet finite when all the Goldstone boson interactions to the order considered are included. Whereas we recover previous results for $\mu \rightarrow e\gamma$ [12, 13].

The present limits on the rates of LFV processes translate into bounds on the LHT parameters. Tables 1 and 2 show the bounds imposed in the two family case by $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee\bar{e}$, respectively, on the heavy lepton mass splitting δ for a maximal mixing angle, $\sin 2\theta = 1$, and several values of the LH scale f and the ratio \tilde{y} related to the common heavy lepton mass. The main conclusion is that the new parameters must be tuned to 10% for a *natural* value $f \sim 1$ TeV. Obviously, raising f quickly reduces the decay rates, which scale as f^{-4} . The results are also sensitive to the parameter \tilde{y} , but the dependence is mild for moderate values (see figure 8), when rates scale roughly like $\sin^2 2\theta \delta^2$. The non-observation of LFV effects may be also the result of a conspiracy among the new parameters being all slightly above or below their expected natural values, of order one. Analogous fine tuning on the alignment of light and heavy leptons is required in the general case with three families.

In table 3 we give both the present and future bounds if the current limits on $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee\bar{e}$ are improved by two orders of magnitude [40]. An asterisk indicates that the assumed values are excluded for any possible \tilde{y} . A non-empty region for \tilde{y} is recovered

	$\mathcal{B}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ (10^{-13})	$\mathcal{B}(\mu \rightarrow ee\bar{e}) < 10^{-12}$ (10^{-14})
$f/\text{TeV} >$	2.5 (8.1)	2.3 (7.4)
$\sin 2\theta <$	0.16 (0.015)	0.16 (0.016)
$ \delta <$	0.16 (0.015)	0.18 (0.018)
$\tilde{y} <$	0.16 (*)	* (*)

Table 3: Bounds from current (future) experiments on individual LHT parameters, assuming the others fixed to the *natural* values $f = 1 \text{ TeV}$, $\sin 2\theta = \delta = \tilde{y} = 1$. An asterisk means that the quoted limit on the branching ratio excludes any \tilde{y} value for the assumed values of the other parameters.

increasing f or decreasing $\sin 2\theta$ and/or δ (see figures 6 and 7). Finally, we must note that the limits on the corresponding tau decays $\tau \rightarrow \mu\gamma, e\gamma$ and $\tau \rightarrow \mu\mu\bar{\mu}, ee\bar{e}$ are weaker [43], typically $< 10^{-8} - 10^{-7}$. Then, they do not further restrict the order parameter f for natural values of the other heavy lepton parameters, but could eventually constrain the corresponding mixing angles and heavy lepton masses, which are in principle independent of the parameters otherwise involved in the muon to electron processes. However, present limits give no significative bound on the parameters related to the third lepton family.

Note added. During the completion of this manuscript several related papers were released. The one-loop contributions in the LHT to the tbW vertex in ref. [44] and to $Z\ell\ell'$ in ref. [45] have been calculated. In neither case has the order v^2/f^2 correction to the SM weak boson coupling to heavy right-handed fermions been included. More recently a new analysis of B decays in the this model has been carried out in ref. [46], yielding an ultraviolet finite result when this correction was taken into account following ref. [15] and in agreement with our findings.

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A. Physical fields

After the electroweak symmetry breaking (EWSB) the SM gauge boson mass eigenstates, which are the T-even, write

$$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2), \quad \begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}, \quad (\text{A.1})$$

with

$$W^a = \frac{W_1^a + W_2^a}{\sqrt{2}}, \quad B = \frac{B_1 + B_2}{\sqrt{2}}; \quad (\text{A.2})$$

whereas the T-odd combinations expanding the heavy sector read to order v^2/f^2 are

$$W_H^\pm = \frac{1}{\sqrt{2}}(W_H^1 \mp iW_H^2), \quad \begin{pmatrix} Z_H \\ A_H \end{pmatrix} = \begin{pmatrix} 1 & -x_H \frac{v^2}{f^2} \\ x_H \frac{v^2}{f^2} & 1 \end{pmatrix} \begin{pmatrix} W_H^3 \\ B_H \end{pmatrix}, \quad (\text{A.3})$$

with

$$W_H^a = \frac{W_1^a - W_2^a}{\sqrt{2}}, \quad B_H = \frac{B_1 - B_2}{\sqrt{2}}, \quad x_H = \frac{5gg'}{4(5g^2 - g'^2)}. \quad (\text{A.4})$$

Their masses to order v^2/f^2 are

$$\begin{aligned} M_W &= \frac{gv}{2} \left(1 - \frac{v^2}{12f^2}\right), & M_Z &= M_W/c_W, & e &= g_{SW} = g'c_W, & v &\simeq 246 \text{ GeV}, \\ M_{W_H} &= M_{Z_H} = gf \left(1 - \frac{v^2}{8f^2}\right), & M_{A_H} &= \frac{g'f}{\sqrt{5}} \left(1 - \frac{5v^2}{8f^2}\right). \end{aligned} \quad (\text{A.5})$$

The scalar fields must be also rotated into the physical fields [20]:

$$\pi^0 \rightarrow \pi^0 \left(1 + \frac{v^2}{12f^2}\right), \quad (\text{A.6})$$

$$\pi^\pm \rightarrow \pi^\pm \left(1 + \frac{v^2}{12f^2}\right), \quad (\text{A.7})$$

$$h \rightarrow h, \quad (\text{A.8})$$

$$\Phi^0 \rightarrow \Phi^0 \left(1 + \frac{v^2}{12f^2}\right), \quad (\text{A.9})$$

$$\Phi^P \rightarrow \Phi^P + \left(\sqrt{10}\eta - \sqrt{2}\omega^0 + \Phi^P\right) \frac{v^2}{12f^2}, \quad (\text{A.10})$$

$$\Phi^\pm \rightarrow \Phi^\pm \left(1 + \frac{v^2}{24f^2}\right) \pm i\omega^\pm \frac{v^2}{12f^2}, \quad (\text{A.11})$$

$$\Phi^{++} \rightarrow \Phi^{++}, \quad (\text{A.12})$$

$$\eta \rightarrow \eta + \frac{5g'\eta - 4\sqrt{5}[g'(\omega^0 + \sqrt{2}\Phi^P) - 6gx_H\omega^0] v^2}{24g' f^2}, \quad (\text{A.13})$$

$$\omega^0 \rightarrow \omega^0 + \frac{5g(\omega^0 + 4\sqrt{2}\Phi^P) - 4\sqrt{5}\eta(5g + 6g'x_H) v^2}{120g f^2}, \quad (\text{A.14})$$

$$\omega^\pm \rightarrow \omega^\pm \left(1 + \frac{v^2}{24f^2}\right) \pm i\Phi^\pm \frac{v^2}{f^2}. \quad (\text{A.15})$$

For each SM left-handed lepton doublet there is an extra vector-like doublet,

$$l_{iL} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}, \quad i = 1, 2, \quad l_{HR} = \begin{pmatrix} \nu_{HR} \\ \ell_{HR} \end{pmatrix}. \quad (\text{A.16})$$

Then the left-handed mass eigenstates are

$$l_L = \frac{l_{1L} - l_{2L}}{\sqrt{2}}, \quad l_{HL} = \frac{l_{1L} + l_{2L}}{\sqrt{2}}, \quad l = \nu, \ell, \quad (\text{A.17})$$

where we omit the flavour index. ν_L, ℓ_L are the SM (T-even) left-handed leptons, whereas ν_{HL}, ℓ_{HL} (ν_{HR}, ℓ_{HR}) are T-odd left (right) handed leptons with masses of $\mathcal{O}(f)$. The SM right-handed fermions are assumed to be singlets under the non-abelian symmetries. Heavy leptons receive their masses from the Yukawa term proportional to κ (2.19), which is in general a non-diagonal matrix in flavour space that induces flavour mixing in the T-odd sector. The misalignment between the mass matrices of the T-even (SM) and T-odd (heavy) sectors is a source of intergeneration mixing (see section 2.2). The diagonalisation of the κ matrix (see eq. (2.30)) yield the heavy lepton masses

$$m_{\ell_H^i} = \sqrt{2}\kappa_{ii}f \equiv m_{Hi}, \quad m_{\nu_H^i} = m_{Hi} \left(1 - \frac{v^2}{8f^2}\right). \quad (\text{A.18})$$

B. Feynman rules

We present below just the Feynman rules which are necessary for the calculation of the LFV processes discussed in this work. They are given in terms of generic couplings for the following general vertices involving scalars (S), fermions (F) and/or gauge bosons (V):

$$[\text{V}_\mu \text{FF}] = ie\gamma^\mu(g_L P_L + g_R P_R), \quad (\text{B.1})$$

$$[\text{SFF}] = ie(c_L P_L + c_R P_R), \quad (\text{B.2})$$

$$[\text{SV}_\mu \text{V}_\nu] = ieK g^{\mu\nu}, \quad (\text{B.3})$$

$$[\text{V}_\mu \text{S}(p_1)\text{S}(p_2)] = ieG(p_1 - p_2)^\mu, \quad (\text{B.4})$$

$$[\text{V}_\mu(p_1)\text{V}_\nu(p_2)\text{V}_\rho(p_3)] = ieJ [g^{\mu\nu}(p_2 - p_1)^\rho + g^{\nu\rho}(p_3 - p_2)^\mu + g^{\mu\rho}(p_1 - p_3)^\nu], \quad (\text{B.5})$$

where all momenta are assumed incoming. The conjugate vertices are obtained replacing:

$$g_{L,R} \leftrightarrow g_{L,R}^*, \quad c_{L,R} \leftrightarrow c_{L,R}^*, \quad K \leftrightarrow K^*, \quad G \leftrightarrow G^*, \quad J \leftrightarrow J^*. \quad (\text{B.6})$$

B.1 SM with massive neutrinos

For comparison we first give the rules for the SM with light massive neutrinos. The sign conventions for the covariant derivatives are those in ref. [22].

VFF	$\gamma \bar{f}^i f^j$	$Z \bar{f}^i f^j$	$W^+ \bar{\nu}^i \ell^j$	$W^- \bar{\ell}^j \nu^i$
g_L	$-Q_f \delta_{ij}$	$Z_L^f \delta_{ij}$	$\frac{1}{\sqrt{2}s_W} V_{\text{PMNS}}^{ji*}$	$\frac{1}{\sqrt{2}s_W} V_{\text{PMNS}}^{ji}$
g_R	$-Q_f \delta_{ij}$	$Z_R^f \delta_{ij}$	0	0

where $Z_{L,R}^f = (v_f \pm a_f)/2s_W c_W$ with $v_f = T_3^{fL} - 2Q_f s_W^2$ and $a_f = T_3^{fL}$.

SFF	$\phi^+ \bar{\nu}^i \ell^j$
c_L	$+\frac{1}{\sqrt{2}s_W} \frac{m_{\nu^i}}{M_W} V_{\text{PMNS}}^{ji*}$
c_R	$-\frac{1}{\sqrt{2}s_W} \frac{m_{\ell^j}}{M_W} V_{\text{PMNS}}^{ji*}$

SVV	$\phi^\pm W^\mp \gamma$	$\phi^\pm W^\mp Z$
K	$-M_W$	$-M_W s_W / c_W$

VSS	$\gamma\phi^\pm\phi^\mp$	$Z\phi^\pm\phi^\mp$
G	∓ 1	$\pm \frac{c_W^2 - s_W^2}{2s_W c_W}$

VVV	$\gamma W^+ W^-$	$Z W^+ W^-$
J	-1	c_W/s_W

The fields ϕ^\pm are the would-be Goldstone bosons eaten by the gauge bosons fields W^\pm after the EWSB.

B.2 LHT model

The sign conventions are chosen to be compatible with those employed for the SM (which coincide with those in [26] up to a sign in the definition of the abelian gauge couplings in the covariant derivative in eq. (2.14)). In particular, these Feynman rules include the $\mathcal{O}(v^2/f^2)$ contribution to the $Z\bar{\nu}_H^i\nu_H^j$ vertex missed in the literature.

VFF	$\gamma\bar{f}_H^i f_H^j$	$Z\bar{\nu}_H^i\nu_H^j$	$Z\bar{\ell}_H^i\ell_H^j$
g_L	$-Q_f\delta_{ij}$	$\frac{1}{2s_W c_W}\delta_{ij}$	$\frac{1}{2s_W c_W}(-1 + 2s_W^2)\delta_{ij}$
g_R	$-Q_f\delta_{ij}$	$\frac{1}{2s_W c_W}\left(1 - \frac{v^2}{4f^2}\right)\delta_{ij}$	$\frac{1}{2s_W c_W}(-1 + 2s_W^2)\delta_{ij}$

VFF	$A_H\bar{\ell}_H^i\ell_H^j$	$Z_H\bar{\ell}_H^i\ell_H^j$	$W_H^+\bar{\nu}_H^i\ell_H^j$
g_L	$\left(\frac{1}{10c_W} - \frac{x_H v^2}{2s_W f^2}\right)V_{H\ell}^{ij}$	$-\left(\frac{1}{2s_W} + \frac{x_H v^2}{10c_W f^2}\right)V_{H\ell}^{ij}$	$\frac{1}{\sqrt{2}s_W}V_{H\ell}^{ij}$
g_R	0	0	0

SFF	$\eta\bar{\ell}_H^i\ell_H^j$	$\omega^0\bar{\ell}_H^i\ell_H^j$
c_L	$\frac{i}{10c_W} \frac{m_{\ell_H^i}}{M_{A_H}} \left[1 - \frac{v^2}{f^2} \left(\frac{5}{4} + x_H \frac{s_W}{c_W}\right)\right] V_{H\ell}^{ij}$	$\frac{i}{2s_W} \frac{m_{\ell_H^i}}{M_{Z_H}} \left[1 + \frac{v^2}{f^2} \left(-\frac{1}{4} + x_H \frac{c_W}{s_W}\right)\right] V_{H\ell}^{ij}$
c_R	$-\frac{i}{10c_W} \frac{m_{\ell^i}}{M_{A_H}} V_{H\ell}^{ij}$	$-\frac{i}{2s_W} \frac{m_{\ell^i}}{M_{Z_H}} V_{H\ell}^{ij}$

SFF	$\omega^+\bar{\nu}_H^i\ell_H^j$
c_L	$-\frac{i}{\sqrt{2}s_W} \frac{m_{\nu_H^i}}{M_{W_H}} V_{H\ell}^{ij}$
c_R	$\frac{i}{\sqrt{2}s_W} \frac{m_{\ell^i}}{M_{W_H}} V_{H\ell}^{ij}$

SVV	$\omega^\pm W_H^\mp \gamma$	$\omega^\pm W_H^\mp Z$
K	$\pm i M_{W_H}$	$\mp i M_{W_H} \frac{c_W}{s_W} \left(1 - \frac{v^2}{4f^2 c_W^2}\right)$

VSS	$\gamma\omega^\pm\omega^\mp$	$Z\omega^\pm\omega^\mp$
G	∓ 1	$\pm \frac{c_W}{s_W} \left(1 - \frac{v^2}{8f^2 c_W^2}\right)$

VVV	$\gamma W_H^+ W_H^-$	$Z W_H^+ W_H^-$
J	-1	c_W/s_W

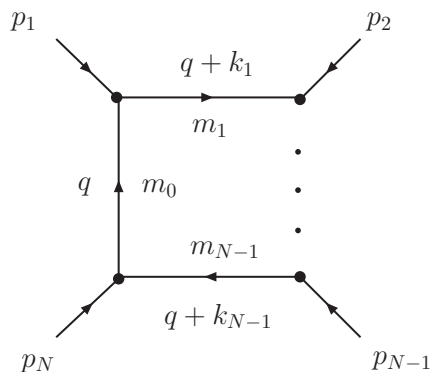


Figure 9: Generic one-loop diagram with N legs.

The fields ω^\pm , ω^0 and η are the Goldstone bosons of the $[\text{SU}(2) \times \text{U}(1)]_1 \times [\text{SU}(2) \times \text{U}(1)]_2$ breaking into its diagonal subgroup. They are eaten by the heavy gauge bosons W_H^\pm , Z_H and A_H , respectively. (Actually these Goldstone bosons mix with an additional physical Higgs triplet Φ at order v^2/f^2 and it is this linear combination of fields that is eaten.) In principle, also the scalar triplet Φ contributes to the processes considered here. The corresponding diagrams can be obtained replacing W_H^\pm by Φ^\pm and Z_H, A_H by Φ^0 and Φ_P^0 . The Feynman rules for the vertices containing Φ , neglecting the masses of the SM fermions, involve couplings of $\mathcal{O}(v^2/f^2)$. As each diagram contains at least two such vertices, if any, they are suppressed by a factor of $\mathcal{O}(v^4/f^4)$ [26].

C. Loop integrals

Consider the generic one-loop diagram with N legs in figure 9, where

$$k_1 = p_1, \quad k_2 = p_1 + p_2, \quad \dots \quad k_{N-1} = \sum_{i=1}^{N-1} p_i. \quad (\text{C.1})$$

This diagram involves integrals of the type

$$\frac{i}{16\pi^2} T_{\mu_1 \dots \mu_P}^N \equiv \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{q_{\mu_1} \dots q_{\mu_P}}{(q^2 - m_0^2) [(q+k_1)^2 - m_1^2] \dots [(q+k_{N-1})^2 - m_{N-1}^2]}. \quad (\text{C.2})$$

These integrals are symmetric under permutation of the Lorentz indices. The integration is performed in dimensional regularization. The mass scale μ keeps track of the correct dimension of the integral in $D = 4 - \epsilon$ spacetime dimensions. $P \leq N$ is the number of q 's in the numerator and determines the tensor structure of the integral (scalar for $P = 0$, vector for $P = 1$, etc.) The notation is A for T^1 , B for T^2 , etc. and the scalar integrals are A_0, B_0 , etc. The tensor integrals can be decomposed into a linear combination of Lorentz covariant tensors constructed from $g_{\mu\nu}$ and a linearly independent set of the momenta [47]. The choice of the basis is not unique. Here we choose $g_{\mu\nu}$ and the momenta k_i , which are sums of the external momenta p_i [22]. In this basis, the tensor-coefficient functions are

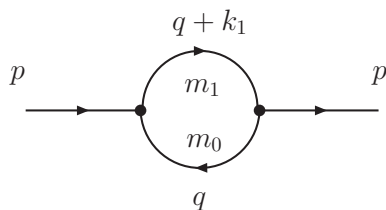


Figure 10: Generic one-loop diagram with two legs.

totally symmetric in their indices. For this work, we need the following decompositions:

$$B_\mu = k_{1\mu} B_1, \quad (\text{C.3})$$

$$C_\mu = k_{1\mu} C_1 + k_{2\mu} C_2, \quad (\text{C.4})$$

$$C_{\mu\nu} = g_{\mu\nu} C_{00} + \sum_{i,j=1}^2 k_{i\mu} k_{j\nu} C_{ij}, \quad (\text{C.5})$$

$$D_\mu = \sum_{i=1}^3 k_{i\mu} D_i, \quad (\text{C.6})$$

$$D_{\mu\nu} = g_{\mu\nu} D_{00} + \sum_{i,j=1}^3 k_{i\mu} k_{j\nu} D_{ij}. \quad (\text{C.7})$$

These functions have been calculated for the argument configuration required by the processes under study, obtaining the following results.

C.1 Two-point functions

Consider now the diagram with two legs in figure 10:

$$\frac{i}{16\pi^2} \{B_0, B^\mu\}(\text{args}) = \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{\{1, q^\mu\}}{(q^2 - m_0^2) [(q+p)^2 - m_1^2]}, \quad (\text{C.8})$$

where $k_1 = p$. The corresponding tensor coefficients are functions of the invariant quantities $(\text{args}) = (p^2, m_0^2, m_1^2)$. The functions $B \equiv B(0; M_1^2, M_2^2)$ and $\bar{B} \equiv B(0; M_2^2, M_1^2)$ read

$$B_0 = \bar{B}_0 = \Delta_\epsilon + 1 - \frac{M_1^2 \ln \frac{M_1^2}{\mu^2} - M_2^2 \ln \frac{M_2^2}{\mu^2}}{M_1^2 - M_2^2}, \quad (\text{C.9})$$

$$\begin{aligned} B_1 &= -\frac{\Delta_\epsilon}{2} + \frac{4M_1^2 M_2^2 - 3M_1^4 - M_2^4 + 2M_1^4 \ln \frac{M_1^2}{\mu^2} + 2M_2^2 (M_2^2 - 2M_1^2) \ln \frac{M_2^2}{\mu^2}}{4(M_1^2 - M_2^2)^2} \\ &= -\bar{B}_0 - \bar{B}_1, \end{aligned} \quad (\text{C.10})$$

with $\Delta_\epsilon \equiv \frac{2}{\epsilon} - \gamma + \ln 4\pi$. These functions are ultraviolet divergent in $D = 4$ dimensions.

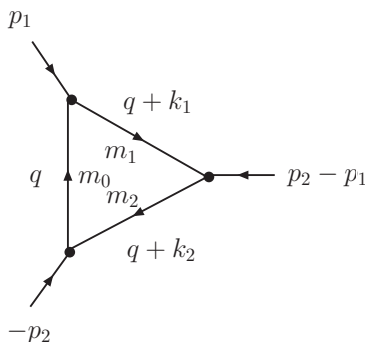


Figure 11: Generic one-loop diagram with three legs.

C.2 Three-point functions

Consider now the diagram with three legs in figure 11:

$$\frac{i}{16\pi^2} \{C_0, C^\mu, C^{\mu\nu}\}(\text{args}) = \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{\{1, q^\mu, q^\mu q^\nu\}}{(q^2 - m_0^2) [(q + p_1)^2 - m_1^2] [(q + p_2)^2 - m_2^2]}, \quad (\text{C.11})$$

where we have chosen the external momenta so that $k_1 = p_1$, $k_2 = p_2$. The corresponding tensor coefficients depend on the invariant quantities $(\text{args}) = (p_1^2, Q^2, p_2^2; m_0^2, m_1^2, m_2^2)$, with $Q^2 \equiv (p_2 - p_1)^2$. The functions $C \equiv C(0, Q^2, 0; M_1^2, M_2^2, M_2^2)$ with $x \equiv M_2^2/M_1^2$ read

$$C_0 = \frac{1}{M_1^2} \left[\frac{1 - x + \ln x}{(1 - x)^2} + \frac{Q^2}{M_1^2} \frac{-2 - 3x + 6x^2 - x^3 - 6x \ln x}{12x(1 - x)^4} \right] + \mathcal{O}(Q^4), \quad (\text{C.12})$$

$$C_1 = C_2 = \frac{1}{M_1^2} \frac{-3 + 4x - x^2 - 2 \ln x}{4(1 - x)^3} + \mathcal{O}(Q^2), \quad (\text{C.13})$$

$$C_{11} = C_{22} = 2 C_{12} = \frac{1}{M_1^2} \frac{11 - 18x + 9x^2 - 2x^3 + 6 \ln x}{18(1 - x)^4} + \mathcal{O}(Q^2), \quad (\text{C.14})$$

$$C_{00} = -\frac{1}{2} B_1 - \frac{Q^2}{M_1^2} \frac{11 - 18x + 9x^2 - 2x^3 + 6 \ln x}{72(1 - x)^4} + \mathcal{O}(Q^4). \quad (\text{C.15})$$

Or else, defining $\bar{C} \equiv C(0, Q^2, 0; M_2^2, M_1^2, M_1^2)$,

$$\bar{C}_0 = \frac{1}{M_1^2} \left[\frac{-1 + x - x \ln x}{(1 - x)^2} + \frac{Q^2}{M_1^2} \frac{-1 + 6x - 3x^2 - 2x^3 + 6x^2 \ln x}{12(1 - x)^4} \right] + \mathcal{O}(Q^4), \quad (\text{C.16})$$

$$\bar{C}_1 = \bar{C}_2 = \frac{1}{M_1^2} \frac{1 - 4x + 3x^2 - 2x^2 \ln x}{4(1 - x)^3}, \quad (\text{C.17})$$

$$\bar{C}_{11} = \bar{C}_{22} = 2 \bar{C}_{12} = \frac{1}{M_1^2} \frac{-2 + 9x - 18x^2 + 11x^3 - 6x^3 \ln x}{18(1 - x)^4}, \quad (\text{C.18})$$

$$\overline{C}_{00} = -\frac{1}{2}\overline{B}_1 - \frac{Q^2}{M_1^2} \frac{-2 + 9x - 18x^2 + 11x^3 - 6x^3 \ln x}{72(1-x)^4} + \mathcal{O}(Q^4). \quad (\text{C.19})$$

Note that C_{00} and \overline{C}_{00} are ultraviolet divergent in $D = 4$ dimensions.

In the limit $Q^2 = 0$ the following useful relations among two- and three-point functions hold:

$$\overline{B}_1 + 2\overline{C}_{00} = 0, \quad (\text{C.20})$$

$$-\frac{1}{4} + \frac{1}{2}\overline{B}_1 + C_{00} - \frac{x}{2}M_1^2 C_0 = 0, \quad (\text{C.21})$$

$$-\frac{1}{2} + \overline{B}_1 + 6\overline{C}_{00} - xM_1^2 \overline{C}_0 = \Delta_\epsilon - \ln \frac{M_1^2}{\mu^2}. \quad (\text{C.22})$$

C.3 Four-point functions

The ones we need are all ultraviolet finite:

$$\frac{i}{16\pi^2} \{D_0, D^\mu, D^{\mu\nu}\}(\text{args}) = \int \frac{d^4 q}{(2\pi)^4} \frac{\{1, q^\mu, q^\mu q^\nu\}}{(q^2 - m_0^2) [(q + k_1)^2 - m_1^2] [(q + k_2)^2 - m_2^2] [(q + k_3)^2 - m_3^2]}, \quad (\text{C.23})$$

with $k_j = \sum_{i=1}^j p_i$ and $(\text{args}) = (p_1^2, p_2^2, p_3^2, p_4^2, (p_1 + p_2)^2, (p_2 + p_3)^2; m_0^2, m_1^2, m_2^2, m_3^2)$. In the limit of zero external momenta, only the following integrals are relevant:

$$\frac{i}{16\pi^2} D_0 = \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_0^2) (q^2 - m_1^2) (q^2 - m_2^2) (q^2 - m_3^2)}, \quad (\text{C.24})$$

$$\frac{i}{16\pi^2} D_{00} = \frac{1}{4} \int \frac{d^4 q}{(2\pi)^4} \frac{q^2}{(q^2 - m_0^2) (q^2 - m_1^2) (q^2 - m_2^2) (q^2 - m_3^2)}. \quad (\text{C.25})$$

In terms of the mass ratios $x = m_1^2/m_0^2$, $y = m_2^2/m_0^2$, $z = m_3^2/m_0^2$ the integrals above can be written as:

$$d_0(x, y, z) \equiv m_0^4 D_0 = \left[\frac{x \ln x}{(1-x)(x-y)(x-z)} - \frac{y \ln y}{(1-y)(x-y)(y-z)} + \frac{z \ln z}{(1-z)(x-z)(y-z)} \right], \quad (\text{C.26})$$

$$\tilde{d}_0(x, y, z) \equiv 4m_0^2 D_{00} = \left[\frac{x^2 \ln x}{(1-x)(x-y)(x-z)} - \frac{y^2 \ln y}{(1-y)(x-y)(y-z)} + \frac{z^2 \ln z}{(1-z)(x-z)(y-z)} \right]. \quad (\text{C.27})$$

For two equal masses ($m_0 = m_3$) we get

$$d_0(x, y) = - \left[\frac{x \ln x}{(1-x)^2(x-y)} - \frac{y \ln y}{(1-y)^2(x-y)} + \frac{1}{(1-x)(1-y)} \right], \quad (\text{C.28})$$

$$\tilde{d}_0(x, y) = - \left[\frac{x^2 \ln x}{(1-x)^2(x-y)} - \frac{y^2 \ln y}{(1-y)^2(x-y)} + \frac{1}{(1-x)(1-y)} \right]. \quad (\text{C.29})$$

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